

# The Maths Tutor

Book 1: Number — KS3 · Grades 1–4

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Jacqueline Biggs

STUDENT EDITION

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## Chapter 1

# Introduction

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This book covers all the *Number* topics needed for maths grades 2 to 4. Each topic has a maths level associated with it. For example, the grades 2 to 3 topics are more suitable if you are in years 7 and 8. The grades 3 to 4 topics are more appropriate if you are in year 9. The key points for each topic are highlighted in blue. A help section is included that is grade 1 to 2 in difficulty.

The book is designed so that you can start anywhere and depending on what you need help with, you may be directed somewhere else. You can choose a topic, read the explanation and examples then try some questions. If you find the questions too difficult then there is a link to guide you to previous learning. (If you are using an electronic version of the book, you can click on the links). After trying the previous learning questions, you should be able to go back and answer the original questions.

## Chapter 2

# Place Value

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### Grade 3

When we multiply or divide, does the number get larger or smaller? When we multiply we expect the number to get bigger, and when we divide we expect it to get smaller. But what happens if the number is less than one? The opposite happens: when we multiply, the *number gets smaller*, and when we divide, the *number gets bigger*.

#### Key point

Multiplying by a number less than one makes the number smaller; dividing by a number less than one makes it bigger.

#### Example 2.1

$3 \times 2 = 6$	2 is greater than 1, so 3 gets bigger.
$3 \times 0.2 = 0.6$	0.2 is less than 1, so 3 gets smaller.
$6 \div 2 = 3$	2 is greater than 1, so 6 gets smaller.
$6 \div 0.2 = 30$	0.2 is less than 1, so 6 gets bigger.

#### Questions A

Without doing the calculation, say whether the original number gets larger or smaller.

$1.5 \times 4$

$3.3 \times 0.8$

$5.8 \times 7$

$7.2 \times 0.9$

$2.8 \div 2$

$4.10 \div 0.5$

$6.6 \div 0.2$

$8.12 \div 3$

If we already know the result of a calculation, we can work out similar calculations by thinking about the *place value* of the numbers.

#### Definition: Place value

The value of a digit according to its position in the number — its column of units, tens, hundreds, and so on.

#### Example 2.2

Given that  $2.6 \times 4.12 = 10.712$ , are the answers to these calculations bigger or smaller? Find them by comparing with the original.

- (a)  $0.26 \times 4.12$   
 (b)  $2.6 \times 412$   
 (c)  $10.712 \div 26$

- (a) 0.26 is ten times smaller than 2.6 and 4.12 is unchanged, so the answer is ten times smaller:  $0.26 \times 4.12 = 1.0712$ .  
 (b) 412 is a hundred times bigger and 2.6 is unchanged, so the answer is a hundred times bigger:  $2.6 \times 412 = 1071.2$ .  
 (c) Rearranging into a division, 26 is ten times bigger than 2.6, so the answer is ten times smaller:  $10.712 \div 26 = 0.412$ .

### Questions B

Find the answers to these questions by comparing them with the original calculation.

1. Given  $8.57 \times 2.1 = 17.997$ , find: (a)  $857 \times 2.1$ , (b)  $85.7 \times 21$ , (c)  $17.997 \div 21$
2. Given  $2.3 \times 8.45 = 19.435$ , find: (a)  $0.23 \times 8.45$ , (b)  $2.3 \times 84.5$ , (c)  $19.435 \div 0.23$
3. Given  $8.19 \times 3.7 = 30.303$ , find: (a)  $819 \times 0.37$ , (b)  $81.9 \times 0.37$ , (c)  $30.303 \div 0.37$
4. Given  $6.14 \times 7.9 = 48.506$ , find: (a)  $0.614 \times 790$ , (b)  $61.4 \times 0.79$ , (c)  $48.506 \div 790$
5. Given  $8.34 \times 3.7 = 30.858$ , find: (a)  $0.834 \times 3.7$ , (b)  $0.834 \times 37$ , (c)  $30.858 \div 0.37$
6. Given  $15.9 \times 1.2 = 19.08$ , find: (a)  $1.59 \times 120$ , (b)  $159 \times 0.12$ , (c)  $190.8 \div 1.2$

#### Hints / Notes

When you divide by a number that is ten times smaller, the answer gets ten times bigger — it does the opposite.

*Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150); **Inverse Operations** (p. 62)*

## Chapter 3

# Rounding Using Decimal Places and Significant Figures

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To round *down* a number, we simply take off the unwanted digits. To round *up* a number we take off the unwanted digits, then we add 1 to the last digit *we are keeping*. If the first digit to be removed is 5 or more, we round up. If not, we round down.

### Key point

To round *down* a number, take off the unwanted digits. To round *up* a number take off the unwanted digits then add 1 to the last remaining digit.

### Key point

If the first digit to be removed is 5 or more, round up. Otherwise round down.

## 3.1 Rounding to Decimal Places

Grade 2

### Rounding to the Nearest Whole Number

A whole number is a number that has no decimal part, like 34 or 6. To round a decimal to the nearest whole number we take off the decimal point and everything to the right, rounding up or down.

#### Example 3.1 — Round these numbers to the nearest whole number

4.317



Check the first digit to be removed, it is a 3. 3 is less than 5. Round down. Snip off the .317.  
= 4

4.76



Check the first digit to be removed, it is a 7.  
7 is more than 5. Round up. Snip off the .76  
then add 1 to the remaining digit.  
= 5

6.2

Snip off the .2. Round down. = 6

6.7

Snip off the .7. Round up. = 7

19.5

Snip off the .5. Round up. = 20

### Questions A

Round these numbers to the nearest whole number.

- |           |           |
|-----------|-----------|
| 1. 7.3    | 2. 8.2    |
| 3. 10.4   | 4. 29.1   |
| 5. 76.1   | 6. 13.8   |
| 7. 15.6   | 8. 2.5    |
| 9. 3.28   | 10. 6.42  |
| 11. 12.49 | 12. 70.17 |
| 13. 27.82 | 14. 19.17 |
| 15. 10.65 | 16. 42.51 |
| 17. 83.67 | 18. 5.99  |

#### Hints / Notes

Watch out when removing a five. Five always rounds up!

Trouble with this? Jump to: **Rounding to the nearest 10, 100 and 1000** (p. 156)

### Rounding to One Decimal Place

To round a number to one decimal place, we keep one digit to the right of the decimal point, rounding up or down.

#### Example 3.2 — Round these numbers to one decimal place

7.426



Check the first digit to be removed, it is a 2. 2

3.156

is less than 5. Round down. Snip off the 26.

 $= 7.4$ 

3.156



Check the first digit to be removed, it is a 5.

Round up. Snip off the 56 then add 1 to the

last digit. 3.1 becomes 3.2.

 $= 3.2$ 

5.81

Round down.  $= 5.8$ 

11.28

Round up.  $= 11.3$ 

21.25

Round up.  $= 21.3$ 

### Questions B

Round these numbers to one decimal place.

1. 7.31

2. 18.47

3. 10.49

4. 32.45

5. 6.21

6. 87.34

7. 15.65

8. 43.78

9. 33.68

10. 51.16

11. 12.491

12. 36.718

13. 7.526

14. 0.541

15. 0.356

16. 21.255

17. 13.607

18. 41.909

### Rounding to Two Decimal Places

To round a number to 2 decimal places, we keep two digits after the decimal point, rounding up or down.

#### Example 3.3 — Round these numbers to two decimal places

9.263

9.263



We check the first digit to be removed, this is a 3. 3 is less than

5. Round down. Snip off the 3.

 $= 9.26$

3.626

3.626



We check the first digit to be removed, this is a 6. 6 is more than

5. Round up. Snip off the 6 then add 1 to the last remaining digit.

3.62 becomes 3.63.

= 3.63

Round down. = 16.78

Round up. = 82.28

Round up. = 56.26

16.783

82.276

56.255

### Questions C

Round these numbers to two decimal places.

- |            |            |
|------------|------------|
| 1. 7.372   | 2. 28.162  |
| 3. 10.493  | 4. 18.453  |
| 5. 6.813   | 6. 65.245  |
| 7. 35.607  | 8. 25.307  |
| 9. 3.737   | 10. 3.459  |
| 11. 72.447 | 12. 12.406 |
| 13. 7.552  | 14. 57.252 |
| 15. 10.357 | 16. 10.355 |
| 17. 0.989  | 18. 40.499 |

### Questions D

Round each number to the nearest whole number, to 1 decimal place, and to 2 decimal places.

1. 8.937 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
2. 7.393 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
3. 11.365 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
4. 5.859 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
5. 13.675 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
6. 2.959 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
7. 3.648 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
8. 12.374 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
9. 3.709 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
10. 2.570 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
11. 6.993 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places
12. 0.599 (a) to the nearest whole number, (b) to 1 decimal place, (c) to 2 decimal places

## 3.2 Rounding to Significant Figures

### Grade 3

#### Rounding to One Significant Figure

If you are told that you had won £6,470,200 in the National Lottery, then you would probably be most interested in the number 6, the 6 million pounds! Six is the most significant figure, it is the first digit that you come to when reading the number.

When reading a number, the first non-zero digit is the most significant figure. For example, in the number 239000, the 2 is the most significant figure.

We keep the significant figure and take off the unwanted digits by rounding up or down. If the number is a whole number then we replace the removed digits with zeros.

Zeros at the beginning of a number are not significant. For example, in the number 0.0719, the 7 is the most significant figure.

#### Key point

The first non-zero digit is the most significant figure, we take off the rest, rounding up or down.

#### Example 3.4 — Round these numbers to one significant figure

421

4 is the most significant figure. The first digit to be removed is

2. Round down. = 4. Replace the removed digits with zeros.

= 400

26

2 is the most significant figure. The first digit to be removed is

6. Round up. = 3. Replace the removed digit with zero.

= 30

0.00218

2 is the most significant figure. The first digit to be removed is

1. Round down.

= 0.002

2.51

2 is the most significant figure. The first digit to be removed is

5. Round up.

= 3

## Questions

Round these numbers to one significant figure.

- |             |               |
|-------------|---------------|
| 1. 8215     | 2. 0.005277   |
| 3. 5295     | 4. 64         |
| 5. 736.4    | 6. 198.5      |
| 7. 45.72    | 8. 4.38       |
| 9. 3.9181   | 10. 0.031     |
| 11. 6.185   | 12. 0.083     |
| 13. 246.7   | 14. 0.0000372 |
| 15. 407.5   | 16. 54328     |
| 17. 0.03418 | 18. 342.9     |

### Hints / Notes

The answer should have no more than one non-zero digit. If the answer is a whole number, it should be around the same size as before, for example 31 rounds to 30 not to 3! If you don't know how to round up or down, check out rounding using decimal places.

Trouble with this? Jump to: [Rounding to Decimal Places](#) (p. 5)

## Rounding to Two Significant Figures

When reading a number, the first *two* non-zero digits are the most significant. For example, in the number 239100, the 2 and 3 are the most significant figures. In the number 0.0719, the 7 and the 1 are the most significant figures.

We keep the significant figures and take off the unwanted digits by rounding up or down. If the number is a whole number, we replace the removed digits with zeros.

Sometimes zeros are in the middle of a number, for example, the number 0.804 has a zero between the 8 and the 4. This zero *is* significant, it is treated like any other number. To two significant figures the number is 0.80. Another example is the number 0.807, to two significant figures the number is 0.81.

### Key point

The first two non-zero digits are the most significant, we take off the rest, rounding up or down.

### Example 3.5 — Round these numbers to two significant figures

423

4 and 2 are the significant figures. The next digit to be removed is

3. Round down. = 42. Replace the removed digit with zero.

2768

= 420

2 and 7 are the significant figures. The first digit to be removed is

6. Round up. = 28. Replace the removed digits with zeros.

= 2800

4083

4 and 0 are the significant figures (zero is in the middle of the number). The first digit to be removed is 8. Round up. = 41. Replace the removed digits with zeros.

= 4100

0.08352

8 and 3 are the significant figures. The first digit to be removed is

5. Round up.

= 0.084

## Questions

Round these numbers to two significant figures.

- |              |              |
|--------------|--------------|
| 1. 923       | 2. 645       |
| 3. 421       | 4. 0.0238    |
| 5. 738       | 6. 0.174     |
| 7. 26.28     | 8. 0.000987  |
| 9. 3.9281    | 10. 789.3    |
| 11. 6.385    | 12. 49.238   |
| 13. 226.7    | 14. 4.25683  |
| 15. 714.5    | 16. 0.000812 |
| 17. 0.05438  | 18. 4038     |
| 19. 0.003281 | 20. 4083     |

## Hints / Notes

There should be no more than *two* non-zero digits in your answer. If the answer is a whole number, it should be the same size as before, for example 316 rounds to 320 not to 32! If you don't know how to round up or down, check out rounding using decimal places.

*Trouble with this? Jump to: [Rounding to Decimal Places](#) (p. 5)*

## Estimating Using One Significant Figure

Grade 3

When estimating a calculation, we round each number to one significant figure, then we calculate the answer in the usual way.

### Key point

Round each number to one significant figure, then calculate the answer as normal.

### Example 3.6 — Round each number to 1 significant figure, then estimate

$$43.2 \times 31$$

$$482 \times 7.19$$

$$92.1 \div 3.4$$

$$0.632 \div 0.25$$

$$\frac{813}{38.2}$$

$$372 \times (41 - 28)$$

$$\frac{54.8 \times 80.36}{17}$$

$$\frac{0.24 \times 0.568}{0.037}$$

$$40 \times 30 = 1200$$

$$500 \times 7 = 3500$$

$$90 \div 3 = 30$$

$$0.6 \div 0.3 = 2$$

$$\frac{800}{40} = 20$$

$$400 \times (40 - 30) = 4000$$

$$\frac{50 \times 80}{20} = \frac{4000}{20} = \frac{400}{2} = 200$$

$$\frac{0.2 \times 0.6}{0.04} = \frac{0.12}{0.04} = \frac{12}{4} = 3$$

### Questions

Estimate these calculations by first rounding to one significant figure.

1.  $92.1 \times 34$

3.  $282 \times 5.61$

5.  $3.2 \times 25$

7.  $821 \div 1.74$

9.  $12.1 \div 19$

11.  $13.1 \div 4.5$

2.  $\frac{553}{347}$

4.  $\frac{853}{347}$

6.  $3.81 \times (71 - 12)$

8.  $\frac{84.8 \times 2.36}{17}$

10.  $\frac{6.2 \times 1.68}{37}$

12.  $0.91 \times 85.5$

### Hints / Notes

Each number should have only *one* non-zero digit before working out the answer.

*Trouble with this? Jump to: **Rounding to One Significant Figure** (p. 9); **BIDMAS** (p. 60)*

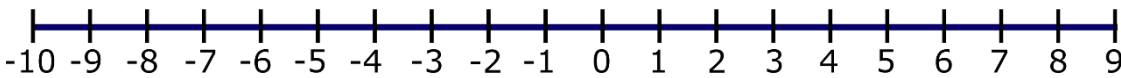
## Chapter 4

# Negative Numbers

## 4.1 Ordering Negative Numbers

Grade 2

Using the number line below, if we start at zero and move to the right, the numbers are positive and increasing. If instead, we move left, the numbers are negative and they are decreasing. For example,  $-10$  is less than  $-6$ . This helps us to put numbers in order.



### Key point

The more negative the number, the smaller it is.

### Example 4.1

Put these numbers in order smallest to largest.

7,  $-10$ , 5,  $-3$ , 1

The smallest digit, the one that is most to the left on the number line, is  $-10$ , the next smallest is  $-3$  and so on. The order is  $-10$ ,  $-3$ , 1, 5, 7.

### Questions

Put these numbers in order smallest to largest.

1.  $-5$ ,  $-9$ , 3, 0, 8
2.  $-6$ , 12,  $-10$ , 9,  $-4$
3.  $-15$ , 13, 0,  $-7$ , 11
4. 25,  $-18$ ,  $-2$ ,  $-12$ , 7
5. 73,  $-21$ , 57,  $-13$ , 41
6.  $-175$ , 631, 40,  $-722$ , 119
7.  $-6.2$ , 1.2,  $-5.7$ , 9.3,  $-4.8$
8. 2.5,  $-1.8$ ,  $-2$ , 0, 7.6

## 4.2 Adding and Subtracting Negative Numbers

Grade 3

Using the number line below, if we add a positive number we move to the right. If we subtract a positive number, we move to the left.

Temperatures are useful when thinking about negative numbers. If a room has a temperature of  $5^{\circ}\text{C}$  and the temperature increases by  $3^{\circ}\text{C}$ , then the room becomes  $8^{\circ}\text{C}$ .

$$5 + 3 = 8$$

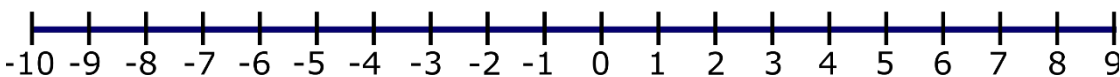
Similarly, if the room has a temperature of  $5^{\circ}\text{C}$  and the temperature decreases by  $6^{\circ}\text{C}$  then the room becomes  $-1^{\circ}\text{C}$ .  $5 - 6 = -1$ .

We can also imagine a lift which has stopped at floor 3. If the lift goes down 5 floors then it would be at floor  $-2$ .  $3 - 5 = -2$ .

If the lift then went up 10 floors, it would be at floor 8.  $-2 + 10 = 8$ .

### Addition

If we add a positive number, we move to the right on the number line. The number gets bigger.



#### Key point

If we add a positive number, we move to the right on the number line.

#### Example 4.2

$-2 + 7 = 5$  Start at  $-2$  on the number line then move 7 places to the right. We end at 5.  
 $-8 + 6 = -2$  Start at  $-8$  on the number line then move 6 places to the right. We end at  $-2$ .

#### Example 4.3

Tom takes his frozen peas out of the freezer; the peas are at a temperature of  $-5^{\circ}\text{C}$ . Tom leaves the peas on the table, they rise in temperature by  $17^{\circ}\text{C}$ . What is the temperature of the peas after Tom leaves them on the table?

Imagine a longer number line. Start at  $-5$  and move 17 places to the right. (Jump 5 to zero then jump 12.) We end at 12.  $-5 + 17 = 12$ . Tom's peas are at  $12^{\circ}\text{C}$ .

### Questions A

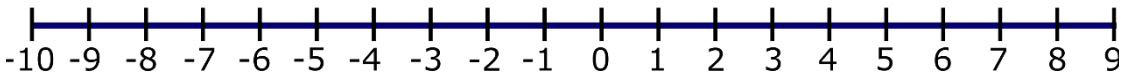
1.  $-3 + 5$
3.  $-2 + 10$
5.  $-8 + 3$
7.  $-6 + 11$
9.  $-7 + 7$

2.  $-9 + 15$
4.  $-1 + 18$
6.  $-5 + 20$
8.  $-6 + 5$
10. Joanna has a tub of ice cream at a temperature of  $-3^{\circ}\text{C}$ , she leaves it on a shelf and

it rises in temperature by  $5^{\circ}\text{C}$ . What is the temperature of the ice cream now?

*Trouble with this? Jump to: **Negative numbers on a Thermometer** (p. 184)*

If we add a negative number, we subtract; we move to the left on the number line. The number gets smaller.



### Key point

If we add a negative number, we move left on the number line.

### Example 4.4

$2 + (-4) = -2$  This is the same as  $2 - 4$ . Start at 2 on the number line and move 4 places to the left. We end at  $-2$ .

$-5 + (-3) = -8$  This is the same as  $-5 - 3$ . Start at  $-5$  on the number line and move 3 places to the left. We end at  $-8$ .

### Example 4.5

In a diving competition, the first dive was awarded 5 marks. The second dive was awarded  $-2$  marks. What was the overall mark?

$5 + (-2)$  — this is the same as  $5 - 2$ . Start at 5 on the number line and move 2 places to the left. We end at 3. The overall mark was 3.

## Questions B

1.  $3 + (-5)$   
 3.  $2 + (-6)$   
 5.  $7 + (-9)$   
 7.  $5 - 10$

9.  $1 + (-6)$   
 11.  $8 - 9$

2.  $2 + (-2)$   
 4.  $9 - 12$

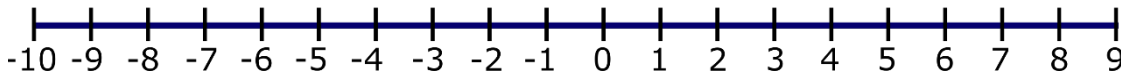
6.  $5 + (-15)$

8. At an ice skating competition, the first Judge awarded 8 marks and the second judge awarded  $-10$  marks. What was the total mark?

10.  $-3 + (-5)$   
 12.  $-8 + (-2)$

## Subtraction

If we subtract a positive number, we move to the left on the number line. The number gets smaller.

**Key point**

If we subtract a positive number, we move left on the number line.

**Example 4.6**

- $6 - 10 = -4$       Start at 6 on the number line and move 10 places to the left. We end at  $-4$ .  
 $8 - 11 = -3$       Start at 8 on the number line and move 11 places to the left. We end at  $-3$ .  
 $-6 - 5 = -11$      Start at  $-6$  on the number line and move 5 places to the left. We end at  $-11$ .

**Example 4.7**

Sonya's room is at a temperature of  $23^{\circ}\text{C}$ . Sonya turns off the heating before she leaves and the temperature goes down by  $25^{\circ}\text{C}$ . What is the temperature of Sonya's room now?

Imagine a longer number line. Start at 23 on the number line and move 25 places to the left. (Jump 23 places to zero then jump 2 places.) We end at  $-2$ .  $23 - 25 = -2$ . Sonya's room is now  $-2^{\circ}\text{C}$ .

Clare likes to go potholing. Clare's current cave is 3m below the surface at  $-3\text{m}$ . Clare moves 5m down into a second cave. What depth is Clare at now?

Start at  $-3$  then move 5 places to the left. We end at  $-8$ .  $-3 - 5 = -8$ . Clare is 8m below the surface.

**Questions C**

- |  |  |
|--|--|
| 1. $5 - 8$   | 2. $-1 - 8$  |
| 3. $3 - 5$   | 4. $-9 - 5$  |
| 5. $2 - 10$  | 6. $-3 - 8$  |
| 7. $7 - 12$  | 8. $-2 - 9$  |
| 9. $13 - 20$   | 10. $-8 - 6$   |
| 11. A lift is currently at floor 5. Then the lift goes down 7 floors. What floor is the lift on now? | 12. $-6 - 11$  |
| 13. $-2 - 4$   | 14. $-5 - 5$   |
| 15. $-5 - 3$   | 16. A diver is currently at a depth of $-20\text{m}$ (20m below sea level). The diver then dives |

down a further 40m. What depth is the diver

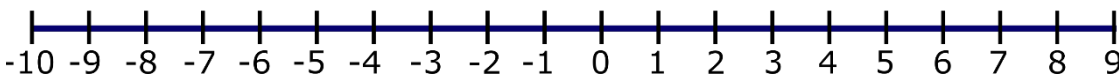


17.  $-7 - 10$

at now?  
18.  $7 - 13$

Trouble with this? Jump to: **Negative numbers on a Thermometer** (p. 184)

If we subtract a negative number, we add. We move to the right on the number line. Subtracting a negative number is the same as adding. Imagine being at the beach, sitting beside a large hole in the sand. To remove the hole, you would *add* sand.



### Key point

If we subtract a negative number, we move right on the number line.

### Example 4.8

$1 - (-5) = 6$       This is the same as  $1 + 5 = 6$ .  
 $4 - (-3) = 7$       This is the same as  $4 + 3 = 7$ .  
 $-7 - (-9) = 2$       This is the same as  $-7 + 9 = 2$ . Start at  $-7$  then move 9 places to the right.  
 We end at 2.

### Example 4.9

Doran was awarded 14 points at a skating competition, including a mark of  $-3$  for lateness. The Judge then took away his lowest mark of  $-3$ . How many points does Doran have now?

$14 - (-3) = 14 + 3 = 17$ . Doran has 17 points.

### Questions D

1.  $5 - (-2)$

3.  $3 - (-8)$

5.  $2 - (-1)$

7.  $7 - (-12)$

9.  $13 - (-10)$

2.  $-3 - (-5)$

4.  $-2 - (-6)$

6.  $-5 - (-10)$

8.  $-6 - (-6)$

10. Lisa got 11 out of 20 for a spelling test. She noticed that a mark of  $-5$  for an incorrect spelling was given in error. She asked for the

$-5$  to be taken away. What mark does she have now?

#### Hints / Notes

$+-$  and  $-+$  become  $-$ . If you have two minus signs next to each other imagine picking one up, turning it, then putting it on top of the other to make a plus!

### Negative Numbers Mixed Questions

- |                |                 |
|----------------|-----------------|
| 1. $3 - 5$     | 2. $-3 + (-5)$  |
| 3. $-2 + 9$    | 4. $-8 - 9$     |
| 5. $8 + (-6)$  | 6. $8 + (-11)$  |
| 7. $-6 - 7$    | 8. $-6 + 15$    |
| 9. $5 - 5$     | 10. $-5 - 12$   |
| 11. $-3 + 2$   | 12. $-3 + 7$    |
| 13. $2 - (-9)$ | 14. $-2 - (+9)$ |
| 15. $-8 + 9$   | 16. $-7 - 4$    |
| 17. $-6 - 11$  | 18. $-2 + 16$   |
| 19. $5 - 8$    | 20. $-3 + 3$    |

## 4.3 Multiplying and Dividing with Negative Numbers

### Grade 3

To multiply or divide with negative numbers, we multiply or divide, ignoring the negative signs at first. Then, if the numbers have the same sign (both numbers are positive or both numbers are negative) then the answer is positive. If the numbers have different signs (one is positive and one is negative), then the answer is negative.

#### Key point

If we multiply or divide numbers with the same sign, the answer is positive. If numbers have different signs, the answer is negative.

#### Example 4.10

- |                     |                                |
|---------------------|--------------------------------|
| $3 \times 4 = 12$   | Both numbers are positive.     |
| $-3 \times -4 = 12$ | Both numbers are negative.     |
| $-3 \times 4 = -12$ | One positive and one negative. |
| $3 \times -4 = -12$ | One positive and one negative. |

### Questions A

- |                   |                    |
|-------------------|--------------------|
| 1. $-3 \times -5$ | 2. $6 \times -7$   |
| 3. $-4 \times -6$ | 4. $-4 \times -8$  |
| 5. $8 \times -5$  | 6. $-1 \times 6$   |
| 7. $-9 \times -4$ | 8. $-10 \times -7$ |

**Hints / Notes**

Negatives like messing everything up. If there is one negative number, then it makes the whole calculation negative. If there are two negatives they cancel out, so the calculation is positive!

**Example 4.11**

$12 \div 6 = 2$	Both numbers are positive.
$-12 \div -6 = 2$	Both numbers are negative.
$-12 \div 6 = -2$	One positive and one negative.
$12 \div -6 = -2$	One positive and one negative.

**Questions B**

- |                  |                  |
|------------------|------------------|
| 1. $-30 \div -5$ | 2. $16 \div -4$  |
| 3. $-24 \div -6$ | 4. $-24 \div 8$  |
| 5. $8 \div -4$   | 6. $-10 \div 10$ |
| 7. $-9 \div -3$  | 8. $-10 \div -2$ |

**Mixed Questions**

- |                   |                     |
|-------------------|---------------------|
| 1. $-8 \times -7$ | 2. $9 \times -11$   |
| 3. $-36 \div -6$  | 4. $-72 \div 8$     |
| 5. $12 \times -4$ | 6. $-10 \times -10$ |
| 7. $-45 \div 9$   | 8. $-10 \times 7$   |

## Chapter 5

# Square and Cube Numbers

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## 5.1 Square Numbers and Square Roots

Grade 3

A square number is a number multiplied by itself.  $1 \times 1$ ,  $2 \times 2$  and  $3 \times 3$  all produce squares.

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

These are the first 4 square numbers but there are many more. From the multiplication table, you can see the square numbers are shown in red along the diagonal line.

$\times$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

One squared is written as  $1^2$ , two squared as  $2^2$  and so on. The first ten square numbers are shown in red as follows:

$$1^2 = 1 \times 1 = 1$$

$$6^2 = 6 \times 6 = 36$$

$$2^2 = 2 \times 2 = 4$$

$$7^2 = 7 \times 7 = 49$$

$$3^2 = 3 \times 3 = 9$$

$$8^2 = 8 \times 8 = 64$$

$$4^2 = 4 \times 4 = 16$$

$$9^2 = 9 \times 9 = 81$$

$$5^2 = 5 \times 5 = 25$$

$$10^2 = 10 \times 10 = 100$$

Any number can be squared, including negative numbers, decimal numbers and fractions.

**Key point**

A square number is a number multiplied by itself.

### Example 5.1

$26^2 = 26 \times 26 = 676$  Multiply 26 by itself using the column method:  $26 \times 6 = 156$  and  $26 \times 20 = 520$ , then  $156 + 520 = 676$ .

### Questions A

Find the values for these square numbers, try not to use a calculator.

1.  $11^2$

3.  $12^2$

5.  $13^2$

7.  $14^2$

9.  $15^2$

2.  $16^2$

4.  $17^2$

6.  $18^2$

8.  $19^2$

10.  $20^2$

### Questions B

Using a calculator, find the values for these square numbers.

1.  $3.2^2$

3.  $(-4)^2$

5.  $103^2$

2.  $1.5^2$

4.  $28^2$

6.  $500^2$

#### Hints / Notes

Can you find the  $x^2$  button on your calculator? What happens to the minus sign when a negative number is squared? It disappears! Look at multiplying whole numbers if you can't remember how.

*Trouble with this? Jump to:* **Multiplying Whole Numbers by Two Digits Column Method** (p. 175); **Multiplying Whole Numbers by Two Digits Table Method** (p. 176)

### Square Roots

The opposite of squaring a number is to square root it. For example, 3 squared is equal to 9, ( $3^2 = 9$ ) so the square root of 9 is 3. This is written as  $\sqrt{9} = 3$ . To find the square root of a number, think about which number multiplies by itself to give you the number in the square root.

#### Key point

Square rooting a number is the opposite of squaring a number.

### Example 5.2

$\sqrt{25} = 5$  Try squaring numbers until the answer is 25:  $2 \times 2 = 4$  (no),  $3 \times 3 = 9$  (no),  $4 \times 4 = 16$  (no),  $5 \times 5 = 25$  (yes!).

## Questions

Find the values of these square roots, use a calculator to check your answers.

1.  $\sqrt{4}$
3.  $\sqrt{36}$
5.  $\sqrt{81}$
7.  $\sqrt{49}$

2.  $\sqrt{16}$
4.  $\sqrt{64}$
6.  $\sqrt{144}$
8.  $\sqrt{100}$

### Hints / Notes

Square numbers help you to work out square roots. If  $6 \times 6 = 36$ , then the square root of 36 is 6. Can you find the  $\sqrt{\quad}$  button on your calculator?

What happens if you try to find the square root of a negative number on a calculator? Try  $\sqrt{-4} =$ . Did you get an error message? That is because you cannot have the square root of a negative number!

## 5.2 Cube Numbers and Cube Roots

Grade 3

### Finding Cube Numbers

A cube or a cubic number is a number multiplied by itself twice. We write one cubed as  $1^3$ , two cubed as  $2^3$  and so on. For example, two cubed is equal to 8, this is because  $2 \times 2 \times 2 = 8$ . This is written as  $2^3 = 8$ .

### Key point

A cubic number is a number multiplied by itself twice.

### Example 5.3

$7^3 = 7 \times 7 \times 7 = 343$  First  $7 \times 7 = 49$ . Then find  $49 \times 7$ : split 49 into 40 plus 9, so  $40 \times 7 = 280$  and  $9 \times 7 = 63$ , and  $280 + 63 = 343$ .

## Questions A

Find the values of these cubes, try not to use a calculator.

1.  $3^3$
3.  $5^3$
5.  $8^3$

2.  $4^3$
4.  $6^3$
6.  $10^3$

### Hints / Notes

The power tells you how many values should be multiplied together.  $2^3 = 2 \times 2 \times$

Trouble with this? Jump to: **Multiplying Whole Numbers by Single Digits** (p. 174)

$2 = 8$ . Don't fall into the trap of thinking that  $2^3$  is the same as  $2 \times 3$ , it isn't!

### Questions B

Using a calculator, find the values of these cubic numbers.

1.  $2.5^3$

3.  $(-6)^3$

5.  $100^3$

2.  $20^3$

4.  $3.8^3$

6.  $0.5^3$

### Hints / Notes

Can you find the  $x^3$  button on your calculator?

### Cube Roots

The opposite of cubing a number is to cube root it. For example, three cubed is equal to 27, ( $3^3 = 27$ ) so the cube root of 27 is three. This is written as  $\sqrt[3]{27} = 3$ . To find the cube root of a number, think about which number multiplies by itself twice to give you the number in the cube root.

### Key point

Cube rooting a number is the opposite of cubing a number.

### Example 5.4

$\sqrt[3]{64} = 4$  Find a number that multiplies twice to make 64. Try 2, 3 then 4:  $2 \times 2 \times 2 = 8$  (no),  $3 \times 3 \times 3 = 27$  (no),  $4 \times 4 \times 4 = 64$  (yes!).

$\sqrt[3]{343} = 7$  343 is a larger number, so start with bigger numbers:  $6 \times 6 \times 6 = 216$  (no, need bigger),  $7 \times 7 \times 7 = 343$  (yes!).

### Questions

Find the value of these cube roots, use a calculator to check your answers.

1.  $\sqrt[3]{1000}$

3.  $\sqrt[3]{125}$

5.  $\sqrt[3]{8}$

7.  $\sqrt[3]{-8}$

2.  $\sqrt[3]{27}$

4.  $\sqrt[3]{1}$

6.  $\sqrt[3]{512}$

8.  $\sqrt[3]{216}$

### Hints / Notes

Can you find the  $\sqrt[3]{\phantom{x}}$  button on your calculator? To find  $\sqrt[3]{64}$  for example, press [Shift] [ $\sqrt{\phantom{x}}$ ] 64 [=]. Did you notice that, unlike square roots, you *can* have a cube root of a negative number!

## Chapter 6

# Indices and Roots

### Grade 3

The index (or plural, indices) shows how many numbers are multiplied together. Another word for index is power. For example,  $3^5$  is three raised to the power five. The index is five.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$ , five lots of three are multiplied together. Another example is  $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$ , six tens are multiplied together.

#### Key point

The index or power shows how many numbers are multiplied together.

### Roots

The root of a number does the opposite of raising a number to a power. Just as the square root of a number does the opposite of squaring it, the fourth root of a number, for example, does the opposite of raising the number to the power 4.

#### Key point

Roots do the opposite of raising a number to a power.

#### Example 6.1

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

$$16, 16 \times 2 = 32, 32 \times 2 = 64.$$

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

$$81, 81 \times 3 = 243.$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10000$$

$$\sqrt[4]{81} = 3$$

(no);  $3 \times 3 \times 3 \times 3 = 81$  (yes!).

Calculating along the line:  $2 \times 2 = 4, 4 \times 2 = 8, 8 \times 2 =$

Calculating along the line:  $3 \times 3 = 9, 9 \times 3 = 27, 27 \times 3 =$

Multiply four tens together.

Try small numbers until one works:  $2 \times 2 \times 2 \times 2 = 16$

### Questions

Find the values of these expressions. Try to use a calculator only when necessary.

1.  $4^4$

3.  $3^5$

5.  $1^9$

7.  $\sqrt[4]{16}$

9.  $\sqrt[5]{243}$

2.  $10^6$

4.  $5^8$

6.  $6^3$

8.  $\sqrt[4]{625}$

10.  $\sqrt[6]{729}$

**Hints / Notes**

To find  $\sqrt[5]{700}$  for example, press [5] [Shift] [ $x^{\square}$ ] 700 [=], you should get 3.70697...

Trouble with this? Jump to: **Square Numbers and Square Roots** (p. 20)

## 6.1 Common Indices or Powers

**Grade 3**

It is useful to learn how certain indices or powers change a number, such as with the indices below.

A number raised to the power of 0 is always 1.  $3^0 = 1$   
 $10^0 = 1$

A number raised to the power of 1 is always itself.  $3^1 = 3$   
 $10^1 = 10$

**Key point**

A number with an index of 0 is 1, a number with an index of 1 is itself.

**Questions**

Write down the values of these numbers.

1.  $5^0$
3.  $17^1$
5.  $64^0$
7.  $35^1$

2.  $9^0$
4.  $100^0$
6.  $0.3^0$
8.  $(-7)^0$

## 6.2 Multiplying and Dividing Numbers with Indices

**Grade 4**

The rules for multiplying and dividing numbers with indices (or powers) are called the Index Laws. If we look at the number  $2^5$ , the large number 2 is called the base number and the small number 5 is called the power or index.

**Multiplying Numbers with Indices**

To multiply numbers with the same base number, raised to a power, we add the powers. Why is that?

For example, if we look at the multiplication  $2^5 \times 2^3$  and if we write out the numbers:

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 \text{ and } 2^3 = 2 \times 2 \times 2$$

$$2^5 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$$

We can instead, simply add the powers.

**Key point**

To multiply the same base numbers raised to a power, add the powers.

**Example 6.2**

$$2^5 \times 2^3 = 2^8 \quad \text{Add the powers: } 5 + 3 = 8.$$

$$3^3 \times 3^4 = 3^7 \quad \text{Add the powers: } 3 + 4 = 7.$$

$$5^6 \times 5^{-4} = 5^2 \quad \text{Add the powers: } 6 + (-4) = 2.$$

**Dividing Numbers with Indices**

To divide numbers with the same base number, raised to a power, we subtract the powers. Why is that?

For example, if we look at the division  $2^5 \div 2^3$  and if we write out the numbers:

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 \quad \text{and} \quad 2^3 = 2 \times 2 \times 2$$

$$2^5 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2^2$$

The 2's above cancel with the 2's below.

We can, instead simply subtract the powers.

**Key point**

To divide the same base numbers raised to a power, subtract the powers.

**Example 6.3**

$$2^5 \div 2^3 = 2^2 \quad \text{Subtract the powers: } 5 - 3 = 2.$$

$$5^9 \div 5^4 = 5^5 \quad \text{Subtract the powers: } 9 - 4 = 5.$$

$$5^{-6} \div 5^4 = 5^{-10} \quad \text{Subtract the powers: } -6 - 4 = -10.$$

**Questions A**

1.  $2^3 \times 2^4$
3.  $5^{10} \div 5^2$
5.  $3^3 \times 3^5$
7.  $8^{10} \times 8^2$
9.  $2^5 \times 2^6$
11.  $5^{10} \div 5^9$
13.  $9^3 \times 9^{-2}$
15.  $3^{10} \div 3^0$
17.  $2^8 \times 2^{-4}$

2.  $2^9 \div 2^7$
4.  $6^5 \times 6^3$
6.  $8^{10} \div 8^3$
8.  $4^5 \times 4^{11}$
10.  $6^5 \div 6^3$
12.  $7^3 \times 7$
14.  $8^9 \div 8^3$
16.  $4^5 \times 4^{-2}$
18.  $9^{10} \div 9$

**Hints / Notes**

What about  $2^4 \times 3^3$ ? You can only add the powers if the base numbers are the same. If they aren't then you must do it the long way... sorry! Think of 7 as  $7^1$  before adding the powers, it's easier!

*Trouble with this? Jump to: **Adding and Subtracting Negative Numbers** (p. 13)*

## A Power Raised to a Power

Sometimes numbers with indices are written inside brackets and the brackets are then raised to a power. This is called a power to a power. For example, if we look at  $(5^3)^4$ , the  $5^3$  is raised to the power 4. For a number with a power, raised to a power, we multiply the powers. Why is that?

For example, if we look at the expression  $(5^3)^4$  and if we write this out:

$$(5^3)^4 = 5^3 \times 5^3 \times 5^3 \times 5^3 = 5^{12}.$$

We can, instead simply multiply the powers.

### Key point

To find a number with a power, raised to a power, multiply the powers.

### Example 6.4

$$\begin{array}{ll} (2^5)^3 = 2^{15} & \text{Multiply the powers: } 5 \times 3 = 15. \\ (2^4)^3 = 2^{12} & \text{Multiply the powers: } 4 \times 3 = 12. \\ (2^5)^{-3} = 2^{-15} & \text{Multiply the powers: } 5 \times (-3) = -15. \end{array}$$

## Questions B

1.  $(5^2)^4$
3.  $(3^3)^2$
5.  $(10^2)^3$
7.  $(6^2)^3$
9.  $(5^{-8})^3$
11.  $(5^2)^5$

2.  $(3^6)^5$
4.  $(4^8)^2$
6.  $(3^4)^2$
8.  $(7^4)^{-2}$
10.  $(3^{-4})^{-2}$
12.  $(3^1)^2$

### Hints / Notes

If you're not sure about this, write the question out the long way, by multiplying the inside expression the number of times shown in the power. For example,  $(5^{-2})^4 = 5^{-2} \times 5^{-2} \times 5^{-2} \times 5^{-2} = 5^{-8}$ .

Trouble with this? Jump to: **Multiplying and Dividing with Negative Numbers** (p. 18)

## Chapter 7

# Reciprocals

### Grade 3

The reciprocal of a number is written as 1 over the number. If you multiply a number by its reciprocal you get 1. For example, the reciprocal of 2 is  $\frac{1}{2}$ . To find the reciprocal of a fraction, flip the fraction. For example, the reciprocal of  $\frac{3}{4}$  is  $\frac{4}{3}$ . The reciprocal of  $\frac{1}{4}$  is  $\frac{4}{1}$  which is 4.

#### Key point

The reciprocal of a number is 1 over the number; the reciprocal of a fraction flips the fraction.

Write the reciprocal of these numbers.

#### Example 7.1

$$8 = \frac{1}{8}$$

Write as 1 over 8.

$$\frac{2}{5} = \frac{5}{2}$$

Flip the fraction.

$$1\frac{1}{8} = \frac{9}{8} = \frac{8}{9}$$

Change the mixed number to a fraction, then flip it.

#### Questions

Write the reciprocal of these numbers.

1. 3

3. 7

5. 9

7. 2

9.  $\frac{1}{6}$

11.  $\frac{6}{1}$

13.  $\frac{5}{10}$

2.  $\frac{1}{2}$

4.  $\frac{1}{7}$

6.  $\frac{1}{9}$

8.  $\frac{1}{10}$

10.  $\frac{1}{7}$

12.  $2\frac{1}{2}$

14.  $3\frac{1}{5}$

#### Hints / Notes

What about the reciprocal of 0?  $\frac{1}{0} = \text{Error!}$

You cannot divide by zero, so there is no reciprocal of 0.

Trouble with this? Jump to: **Changing Mixed Numbers to Improper Fractions** (p. 76)

## Chapter 8

# Divisibility

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Grade 3

We can find out if a number can be divided by certain numbers, without having to do the division.

	A number will divide by:	Example
2	If it is an even number.	Try 317. The last digit, 7, is not even. The number will not divide by 2.
3	If the sum of its digits can be divided by 3.	Try 411. $4 + 1 + 1 = 6$ . 6 can be divided by 3. 411 can be divided by 3.
4	If the last 2 digits can be divided by 4.	Try 316. The number from the last 2 digits, 16, can be divided by 4. 316 can be divided by 4.
5	If the last digit is 0 or 5.	Try 345. The last digit is 5. 345 can be divided by 5.
6	If it can be divided by 2 <u>and</u> 3.	Try 342. The last digit is even; therefore, the number can be divided by 2. $3 + 4 + 2 = 9$ . 9 can be divided by 3. The number can be divided by 2 and 3. 342 can be divided by 6.
9	If the sum of its digits can be divided by 9.	Try 522. $5 + 2 + 2 = 9$ . 9 can be divided by 9. 522 can be divided by 9.
10	If the last digit is 0.	Try 780. The last digit is 0. 780 can be divided by 10.

### Questions

Which of these numbers divide into each number below: 2, 3, 4, 5, 6, 9, 10? Complete the table.

1. 216

2. 114

- 3. 775
- 4. 540
- 5. 198
- 6. 243
- 7. 831
- 8. 528
- 9. 258
- 10. 1328
- 11. 1255
- 12. 9132

## Chapter 9

# Standard Index Form

### Grade 3

Standard index form, or standard form is a short hand way of writing very large or very small numbers by using powers of ten. Numbers like these occur in Physics, Chemistry and Biology. If we wished to write the distance of our Sun from Earth for example, 150,000,000km, we could write this as  $1.5 \times 10^8$ km.

To write a number in standard index form, we write the number between 1 and 10, then we multiply it by 10 to a power. The table below shows powers of ten.

$10^4$	10000
$10^3$	1000
$10^2$	100
$10^1$	10
$10^0$	1
$10^{-1}$	0.1
$10^{-2}$	0.01
$10^{-3}$	0.001

To write 200 in standard index form, for example, we split the number into  $2 \times 100$ . One hundred is  $10^2$ , so the number becomes  $2 \times 10^2$ . To write decimal numbers in standard index form, we count how many spaces the number has moved over the decimal point, this becomes the power of ten.

#### Key point

Write the number between 1 and 10, then multiply by 10 to a power.

#### Example 9.1

$$5400 = 5.4 \times 1000 = 5.4 \times 10^3$$

Split 5400 into  $5.4 \times 1000$ . 1000 is  $10^3$ .

$$716000 = 7.16 \times 100000 = 7.16 \times 10^5$$

Split 716000 into  $7.16 \times 100000$ . 100000 is  $10^5$ .

$$945.6 = 9.456 \times 10^2$$

Write the number between 1 and 10; the first part is

9.456. Move the digits 2 spaces to the right to become 9.456, so the power is 2.

Small numbers have a negative power of 10. To write 0.002 in standard index form, for example, we split the number into  $2 \times 0.001$ . The number 0.001 is  $10^{-3}$ , so the number becomes  $2 \times 10^{-3}$ . We can also use the method of moving digits over the decimal point, the number of spaces moved becomes the power of ten.

Two methods for changing small numbers into standard index form are shown in the examples.

### Example 9.2

$0.000412 = 4.12 \times 0.0001 = 4.12 \times 10^{-4}$  Split 0.000412 into  $4.12 \times 0.0001$ . 0.0001 is  $10^{-4}$ .  
 $0.0005 = 5 \times 10^{-4}$  Write the number between 1 and 10: the first part

is 5. Move the digit 4 spaces to the left to become 5, so the power is  $-4$ .

$0.0096 = 9.6 \times 10^{-3}$  Write the number between 1 and 10: the first part  
 is 9.6. Move the digits 3 spaces to the left to become 9.6, so the power is  $-3$ .

### Questions A

Write the following numbers in standard index form.

- |           |           |
|-----------|-----------|
| 1. 83000  | 2. 200000 |
| 3. 7400   | 4. 7000   |
| 5. 562000 | 6. 9870   |
| 7. 458.3  | 8. 6542.6 |
| 9. 94.38  |           |

#### Hints / Notes

Standard index form is about expressing things in less space. Move the numbers over the decimal point until there's only one non-zero digit in front of it and write  $\times 10^{\text{how many places it moved}}$ .

*Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)*

### Questions B

Write the following numbers in standard index form.

- |              |            |
|--------------|------------|
| 1. 0.0041    | 2. 0.061   |
| 3. 0.00315   | 4. 0.0054  |
| 5. 0.00023   | 6. 0.0073  |
| 7. 0.0000061 | 8. 0.00009 |
| 9. 0.0052    |            |

#### Hints / Notes

Add zeros to the number before moving the digits, it helps to keep the place value of the digits. Remember place value? Hundreds, tens and units?

*Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)*

Sometimes we are given a number in standard index form and we must convert it into a value.

### Method One

Change the power of ten to a number, then multiply it by the first part of the number.

**Example 9.3**

$$1.32 \times 10^3 = 1.32 \times 1000 = 1320$$

Write  $10^3$  as 1000.

$$7.9 \times 10^{-4} = 7.9 \times 0.0001 = 0.00079$$

Write  $10^{-4}$  as 0.0001.**Method Two**

Move the digits over the decimal point the number of times shown in the power. If the power is positive, add zeros to the left of the number before moving the digits, if the power is negative, add zeros to the right of the number before moving.

**Example 9.4**

Write the value of these numbers.

$$6.3 \times 10^5$$

The power is positive, add zeroes to the left of the number: 00000006.3. Keeping the decimal point in position, move the digits 5 places to the left to get 00630000.0. Trim off the extra zeros: = 630000.

$$5.24 \times 10^{-4}$$

The power is negative, add zeros to the right of the number: 5.24000000. Keeping the decimal point in position, move the digits 4 places to the right to get 0.0005240. Trim off the extra zeros: = 0.000524.

The examples showed different methods for changing from standard index form, choose the method that you prefer.

**Questions C**

Write the value of these numbers.

1.  $1.2 \times 10^3$

3.  $8 \times 10^6$

5.  $6.4 \times 10^4$

7.  $4.36 \times 10^{-3}$

9.  $7.12 \times 10^{-5}$

2.  $2 \times 10^4$

4.  $4.3 \times 10^3$

6.  $8.95 \times 10^6$

8.  $6.49 \times 10^{-4}$

When multiplying or dividing numbers in standard index form, we calculate the numbers with no powers first, then we calculate the powers of 10 using the index laws for multiplying or dividing numbers with powers.

There are no quick methods for adding or subtracting numbers in standard index form, so we must change the numbers into values before adding or subtracting.

**Example 9.5**

$(5 \times 10^6) \times (3 \times 10^2) = 15 \times 10^8 = 1.5 \times 10^9$  Split up the calculation:  $5 \times 3 \times 10^6 \times 10^2$ .  
Then  $5 \times 3 = 15$  and  $10^6 \times 10^2 = 10^8$ , giving  $15 \times 10^8$ . Written in standard index form this is  $1.5 \times 10^9$ .

$(8 \times 10^9) \div (2 \times 10^5) = 4 \times 10^4$  Split up the calculation:  $8 \div 2 \times 10^9 \div 10^5$ .  
Then  $8 \div 2 = 4$  and  $10^9 \div 10^5 = 10^4$ .

$(3.5 \times 10^4) + (7.2 \times 10^2) = 3.572 \times 10^4$  This is addition, so write as ordinary numbers  
then add:  $3.5 \times 10^4 = 35000$  and  $7.2 \times 10^2 = 720$ , so  $35000 + 720 = 35720 = 3.572 \times 10^4$ .

## Questions D

Work out the following calculations, leaving your answers in standard index form (no calculator).

1.  $(3 \times 10^2) \times (2 \times 10^3)$

3.  $(2.5 \times 10^3) \times (2 \times 10^5)$

5.  $(12 \times 10^3) \times (3 \times 10^4)$

7.  $(7.1 \times 10^2) + (2.85 \times 10^3)$

2.  $(4 \times 10^3) \times (2 \times 10^4)$

4.  $(4 \times 10^3) \div (2 \times 10^2)$

6.  $(27 \times 10^9) \div (3 \times 10^4)$

8.  $(9.4 \times 10^3) - (2.8 \times 10^3)$

We can use a calculator to work with more difficult numbers. To enter a number in standard index form, use the  $\times 10^x$  key on the calculator. For example, to enter  $5.6 \times 10^7$ , we enter 5.6, press  $\times 10^x$ , then enter 7.

### Example 9.6

Work out the following calculation, leave your answer in standard index form (use a calculator).

$(5.6 \times 10^{23}) \times (3.15 \times 10^{-12})$

Enter 5.6. Press  $\times 10^x$ , enter 23. Press  $[\times]$ .

Enter 3.15. Press  $\times 10^x$ , enter  $-12$ . Press  $[=]$ .

$= 1.764 \times 10^{12}$

## Questions E

Work out the following calculations, leave your answers in standard index form (use a calculator).

1.  $(5.3 \times 10^2) \times (2.4 \times 10^4)$

3.  $(2.16 \times 10^6) \div (4.32 \times 10^{-2})$

2.  $(4.6 \times 10^{-2}) \times (2.4 \times 10^{-5})$

4.  $2.524 \times 10^7 \div 6.31 \times 10^4$

### Hints / Notes

Sometimes the questions have brackets around them and sometimes they don't,  $(5.3 \times 10^2) \times (2.4 \times 10^4)$  is the same as  $5.3 \times 10^2 \times 2.4 \times 10^4$ .

Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)

Sometimes we are given numbers to add or subtract that have *almost* the same power. In these questions, we rewrite one of the numbers so that both numbers have the same power. We then add up the numbers ignoring the power. When we write the answer, we replace the power.

### Example 9.7

Work out the following calculation, leaving your answer in standard index form (no calculator).

$$(4.1 \times 10^{11}) + (3.2 \times 10^{13})$$

Change the number with the smaller power, in this case 11, to match the power of the other number, 13. Divide the number by 100 then add 2 to the power to adjust for this:  $4.1 \times 10^{11} = 0.041 \times 10^{13}$ . The sum becomes  $0.041 \times 10^{13} + 3.2 \times 10^{13}$ . Add the numbers, ignoring the powers as they are now the same:  $0.041 + 3.2 = 3.241$ . Replace the power, checking that the answer is in standard index form:  $= 3.241 \times 10^{13}$ .

### Questions F

Work out the following calculations, leaving your answers in standard index form (no calculator).

1.  $(3.4 \times 10^{12}) + (5.2 \times 10^{13})$

3.  $(9.54 \times 10^{18}) + (1.72 \times 10^{17})$

2.  $(7.962 \times 10^{14}) - (5.8 \times 10^{12})$

4.  $(4.39 \times 10^{23}) - (7.2 \times 10^{21})$

Trouble with this? Jump to: **Standard Index Form** (p. 32)

## Chapter 10

# Prime Numbers

### Grade 3

A prime number is a number that can only be divided by itself and one. One is not a prime number. Two is the first prime number, because two can only be divided by two (itself) and one. Four can be divided by four (itself), one *and two*, so four is not a prime number.

#### Key point

A prime number can only be divided by itself and one.

### Questions A

Using the table below, cross out the number 1 then circle or highlight the number 2. Cross out any numbers that can be divided by 2. Jump to the next number that is not crossed out, this is a 3. Circle or highlight 3, then cross out any remaining numbers that can be divided by 3. Jump to the next number that is not crossed out. Circle or highlight

the number then cross out any numbers that can be divided by this number. Continue in this way until you are left with only the prime numbers. This method is called the sieve of Eratosthenes!

<del>1</del>	2	3	4	5	6	7	8	9	10
11	<del>12</del>	13	<del>14</del>	15	<del>16</del>	17	<del>18</del>	19	20
<del>21</del>	22	23	<del>24</del>	25	<del>26</del>	<del>27</del>	<del>28</del>	29	30
31	<del>32</del>	<del>33</del>	<del>34</del>	35	<del>36</del>	37	<del>38</del>	<del>39</del>	40
41	<del>42</del>	43	<del>44</del>	<del>45</del>	<del>46</del>	47	<del>48</del>	49	50
<del>51</del>	<del>52</del>	53	<del>54</del>	55	<del>56</del>	<del>57</del>	<del>58</del>	59	60
61	<del>62</del>	<del>63</del>	<del>64</del>	<del>65</del>	<del>66</del>	67	<del>68</del>	<del>69</del>	70
71	<del>72</del>	73	<del>74</del>	<del>75</del>	<del>76</del>	77	<del>78</del>	79	80
<del>81</del>	<del>82</del>	83	<del>84</del>	<del>85</del>	<del>86</del>	<del>87</del>	<del>88</del>	89	90
91	<del>92</del>	<del>93</del>	<del>94</del>	<del>95</del>	<del>96</del>	97	<del>98</del>	<del>99</del>	100

1. Which of these numbers are prime numbers?

## 10.1 Testing for Prime Numbers

To test for prime numbers, check to see if the number can be divided by any other prime number. If it can, then the number is *not* a Prime number.

**Example 10.1**

Check to see if this number is a prime number by dividing by the numbers on the top row. Use a calculator if necessary.

	2	3	5	7	11	13	17
217	No	No	No	Yes			

We can stop here because 217 can be divided by 7, therefore it is *not* a prime number.

**Questions B**

Could these numbers be prime numbers? Check by trying to divide each by 2, 3, 5, 7, 11, 13 and 17. Use a calculator if necessary.

1. 323
2. 377
3. 241
4. 143

**Hints / Notes**

The largest prime number found so far has 17 million digits!

It's easier to prove that a number is *not* prime. To prove it is prime we need to try dividing by more numbers.

Prime numbers are used in encryption!

*Trouble with this? Jump to: **Divisibility** (p. 29)*

## Chapter 11

# Factors and Multiples

## 11.1 Factors of a Number

Grade 2

A factor is a number that divides into another number exactly, with no remainder. For example, four is a factor of twelve because  $12 \div 4$  is a whole number. The factors of twelve are 1, 2, 3, 4, 6 and 12.

### Key point

A factor is a number that divides into another number exactly.

### Factor Pairs

Factor pairs are two numbers that multiply to give the original number. Think of one factor of the number, then think of the value to multiply by to get the original number. These two numbers are a factor pair.

#### Example 11.1

**10**

$$1 \times 10 = 10$$

Also

$$2 \times 5 = 10$$

**30**

$$1 \times 30 = 30$$

$$2 \times 15 = 30$$

$$3 \times 10 = 30$$

$$5 \times 6 = 30$$

1 is a factor. 1 multiplied by 10 makes 10. Therefore 10 is the other factor. The factor pair is 1 and 10.

The factor pairs are: 1 and 10, 2 and 5.

The factors are: 1, 2, 5 and 10.

The factor pairs are: 1 and 30, 2 and 15, 3 and 10, 5 and 6.

The factors are: 1, 2, 3, 5, 6, 10, 15 and 30.

### Questions

Find the factor pairs for the following numbers.

1. 24

3. 32

5. 15

7. 52

9. 28

2. 36

4. 40

6. 48

8. 60

10. 42

## 11.2 Prime Factors of a Number

### Grade 3

Prime factors are factors that are also prime numbers. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. Of these, only 2 and 3 are prime numbers. Therefore, the prime factors of 12 are 2 and 3.

#### Key point

The prime factors are factors of a number that are also prime numbers.

#### Example 11.2

Find the factors of 45, then write the prime factors.

The factors are 1, 3, 5, 9, 15 and 45.

The prime factors are 3 and 5.

#### Questions

Find the factors of each number, then write the prime factors.

1. 30
2. 48
3. 70
4. 32
5. 56
6. 24
7. 66
8. 35

#### Hints / Notes

Prime factors are factors that are primes.

*Trouble with this? Jump to: **Factor Spiders** (p. 187); **Prime Numbers** (p. 37)*

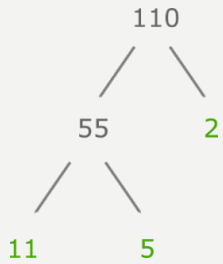
## 11.3 Prime Factors using Factor Trees

### Grade 3

The prime factors of larger numbers can be found by using factor trees. Branches are drawn downwards from the number. Two factors of the number (factor pairs) are written on the end of the branches. This process is repeated until the numbers at the end of the branches are prime numbers. The prime numbers are then written as a product.

#### Key point

The numbers at the end of the branches are prime numbers.

**Example 11.3**

Draw branches down from 110.

$$55 \times 2 = 110.$$

Write 55 and 2 at the end of the branches.

2 is a prime number so the branch stops at 2.

Draw branches down from 55.

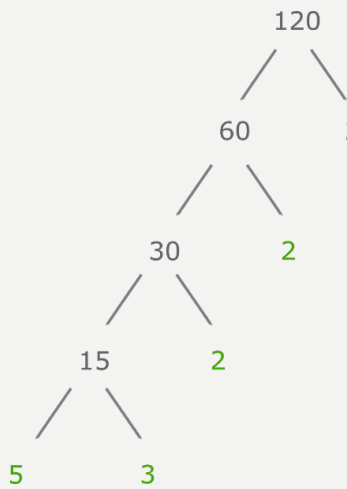
$$11 \times 5 = 55.$$

Write 11 and 5 at the end of these branches.

11 and 5 are prime numbers so the branches stop.

Write the prime numbers as products:

$$110 = 2 \times 11 \times 5$$



Draw branches down from 120.

$$60 \times 2 = 120.$$

Write 60 and 2 at the end of the branches.

2 is a prime number so the branch stops at 2.

Draw branches down from 60.

$$30 \times 2 = 60.$$

Write 30 and 2 at the end of these branches and continue down.

2, 3 and 5 are prime numbers so the branches stop.

Write the prime numbers as products:

$$120 = 2 \times 2 \times 2 \times 5 \times 3 \text{ or } 2^3 \times 5 \times 3$$

**Questions**

Use factor trees to write these numbers as a product of prime factors.

1. 108

3. 136

5. 90

7. 78

2. 450

4. 504

6. 216

8. 143

**Hints / Notes**

It doesn't matter which factor pairs we choose, we end up with the same answer. 'Product' means numbers multiplied together. This is called the Prime Factor Decomposition of a number!  $2 \times 2 \times 2$  is the same as  $2^3$ .

*Trouble with this? Jump to: **Factors of a Number** (p. 39); **Prime Numbers** (p. 37)*

## 11.4 Highest Common Factor (HCF)

### Grade 3

The highest common factor of two numbers is the highest number that is a factor of both numbers. If the factors of both numbers are written in a list, then the highest common factor is the highest number that appears in both lists. The highest common factor is also called the HCF.

#### Key point

The highest common factor is the highest number that is a factor of both numbers.

#### Example 11.4

Find the HCF of 12 and 18.

The factors of 12 are: 1, 2, 3, 4, 6, 12

The factors of 18 are: 1, 2, 3, 6, 9, 18

The HCF is 6.

#### Questions

Find the highest common factor of these numbers.

1. 12 and 20

3. 28 and 32

5. 24 and 60

7. 45 and 75

2. 30 and 75

4. 32 and 40

6. 28 and 56

8. 16 and 36

## 11.5 Multiples of a Number

### Grade 2

Multiples of a number are all the values that the number will divide into, these can be found from your times tables. For example, the multiples of two are the numbers in the two times table: 2, 4, 6, 8, 10 and so on. The first 5 multiples of 3 are: 3, 6, 9, 12, 15.

#### Key point

Multiples are the numbers that your number will divide into.

#### Questions

Write down the first five multiples of the following numbers.

1. 4

3. 8

5. 9

7. 11

2. 5

4. 10

6. 7

8. 15

## 11.6 Lowest Common Multiple (LCM)

Grade 3

The lowest common multiple of two numbers is the smallest number that is a multiple of both numbers. If the multiples of two numbers are written in a list, then the lowest common multiple is the smallest number that is in both lists. The lowest common multiple is also called the LCM.

### Key point

The lowest common multiple is the smallest number that is a multiple of both numbers.

### Example 11.5

Find the lowest common multiple of 6 and 8.

The multiples of 6 are: 6, 12, 18, 24, 30, 36...

The multiples of 8 are: 8, 16, 24, 32, 40...

The lowest common multiple is 24.

### Questions

Find the lowest common multiple of the following numbers.

1. 4 and 5

3. 6 and 9

5. 5 and 7

7. 8 and 6

9. 3 and 12

2. 3 and 7

4. 6 and 4

6. 8 and 10

8. 5 and 9

10. 9 and 8

### Hints / Notes

LCM — the smallest number that both numbers go into.

*Trouble with this? Jump to: **Multiples of a Number** (p. 42)*

## 11.7 HCF and LCM from Prime Factors

Grade 3

We can find the HCF and LCM of two numbers by using a Venn diagram. We find the factors of each number, using a factor tree, then we write down the factors in a list. We identify any common factors (the ones that are in both lists).

To draw a Venn diagram, we draw two overlapping circles. We label one circle with the first number and the second circle with the second number. We write the common factors in the centre, where the circles overlap. We write the remaining factors in the circle belonging to the number. The highest common factor is found from multiplying the factors in the centre. The lowest common multiple is found from multiplying all the factors in the Venn diagram.

**Key point**

The highest common factor is found from multiplying the factors in the centre. The lowest common multiple is found from multiplying all the factors.

**Example 11.6**

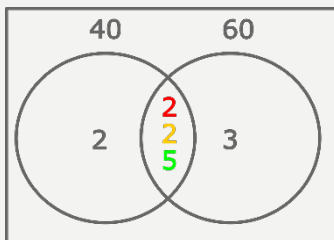
Find the HCF and LCM of 40 and 60.

After drawing a factor tree the prime factors are listed:

$$40 = 2 \times 2 \times 2 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

The common factors are written (once) in the centre. The remaining factors are written in their circles.



HCF — the factors in the centre:

$$2 \times 2 \times 5 = 20$$

LCM — all the factors:

$$2 \times 2 \times 2 \times 5 \times 3 = 120$$

**Example 11.7**

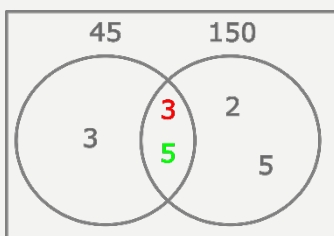
Find the HCF and LCM of 45 and 150.

After drawing a factor tree the prime factors are listed:

$$45 = 3 \times 3 \times 5$$

$$150 = 2 \times 3 \times 5 \times 5$$

Fill in the Venn diagram.



The HCF is  $3 \times 5 = 15$

The LCM is  $3 \times 3 \times 5 \times 2 \times 5 = 450$

## Questions

Find the HCF and LCM of these numbers.

- |                |                |
|----------------|----------------|
| 1. 70 and 28   | 2. 39 and 45   |
| 3. 12 and 66   | 4. 15 and 90   |
| 5. 18 and 24   | 6. 30 and 45   |
| 7. 36 and 48   | 8. 36 and 16   |
| 9. 720 and 252 | 10. 66 and 231 |
| 11. 90 and 105 | 12. 60 and 48  |

### Hints / Notes

Venn Diagrams are explained in the Data Handling Book.

*Trouble with this? Jump to: **Prime Factors using Factor Trees** (p. 40); **Divisibility** (p. 29)*

## Chapter 12

# Doubling and Halving

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### Grade 2

Doubling and halving is a quick way of multiplying numbers mentally. We can sometimes make a multiplication easier, by doubling one number and halving the other. For example, if we need to work out  $3 \times 16$  we can double the 3 and halve the 16. The calculation becomes  $6 \times 8$ , which is 48.

#### Key point

We can make a multiplication easier, by doubling one number and halving the other.

#### Example 12.1

$22 \times 5 = 11 \times 10 = 110$       Halving 22 and doubling 5, the calculation becomes  $11 \times 10$ .  
 $4 \times 16 = 8 \times 8 = 64$       Doubling 4 and halving 16, the calculation becomes  $8 \times 8$ .

#### Questions

Simplify these multiplications, then write the answer.

1.  $14 \times 3$

3.  $8 \times 20$

5.  $4 \times 18$

7.  $3 \times 16$

2.  $18 \times 5$

4.  $16 \times 6$

6.  $5 \times 14$

8.  $2 \times 12$

## Chapter 13

# Multiplying and Dividing with Decimals

Grade 3

## 13.1 Multiplying with Decimal Numbers

To multiply simple decimal numbers, we can treat them as whole numbers and multiply normally. We then adjust the number to take account of the zeros.

### Key point

Treat simple decimal numbers as whole numbers then adjust the answer.

### Example 13.1

$0.7 \times 5 = 3.5$  Ignore the decimal point, multiply 7 by 5:  $7 \times 5 = 35$ . 0.7 is ten times smaller than 7, so divide the answer by ten:  $35 \div 10 = 3.5$ .

$0.8 \times 0.6 = 0.48$  Ignore the decimal point, multiply 8 by 6:  $8 \times 6 = 48$ . 0.8 is ten times smaller than 8, divide by ten:  $48 \div 10 = 4.8$ . 0.6 is ten times smaller than 6, divide by ten again:  $4.8 \div 10 = 0.48$ .

$0.07 \times 2.6 = 0.182$  Ignore the decimal points, multiply 7 by 26:  $7 \times 26 = 182$ . 0.07 is one hundred times smaller than 7, divide by one hundred:  $182 \div 100 = 1.82$ . 2.6 is ten times smaller than 26, divide by ten again:  $1.82 \div 10 = 0.182$ .

### Questions

1.  $0.6 \times 4$
3.  $0.8 \times 7$
5.  $0.9 \times 3$
7.  $0.2 \times 0.4$
9.  $2.7 \times 0.3$
11.  $4.6 \times 0.9$
13.  $1.4 \times 0.5$
15.  $0.35 \times 6$
17.  $0.76 \times 4$
19.  $0.27 \times 0.4$

2.  $0.2 \times 3$
4.  $4 \times 0.8$
6.  $0.8 \times 0.5$
8.  $0.6 \times 0.8$
10.  $9.3 \times 6$
12.  $0.28 \times 7$
14.  $5.8 \times 0.3$
16.  $6.1 \times 0.8$
18.  $0.9 \times 3.7$
20.  $0.8 \times 54$

### Hints / Notes

Ignore the decimal points, do the calculation then adjust the answer.

*Trouble with this? Jump to: **Multiplying Whole Numbers by Single Digits** (p. 174); **Dividing Whole Numbers by Powers of Ten** (p. 151)*

## 13.2 Dividing with Decimal Numbers

To divide simple decimal numbers, we can multiply each number by ten until the number we are dividing by is a whole number. We can then use our times tables, or other multiplications that we know, to find the answer.

Multiply the numbers by ten until they are both whole numbers, then divide.

### Example 13.2

$6 \div 0.3 = 20$       Multiply both numbers by 10:  $6 \times 10 = 60$  and  $0.3 \times 10 = 3$ . The calculation becomes  $60 \div 3 = 20$ .

$1.6 \div 0.4 = 4$       Multiply both numbers by 10:  $1.6 \times 10 = 16$  and  $0.4 \times 10 = 4$ . The calculation becomes  $16 \div 4 = 4$ .

$0.72 \div 0.06 = 12$       Multiply both numbers by 10:  $0.72 \times 10 = 7.2$  and  $0.06 \times 10 = 0.6$ . Still decimals, so repeat:  $7.2 \times 10 = 72$  and  $0.6 \times 10 = 6$ . The calculation becomes  $72 \div 6 = 12$ .

$45 \div 0.5 = 90$       Multiply both numbers by ten:  $450 \div 5$ . We know  $45 \div 5 = 9$ ; 450 is ten times bigger than 45, so the answer is ten times bigger:  $9 \times 10 = 90$ .

$32 \div 0.04 = 800$       Multiply both numbers by ten, then by ten again:  $3200 \div 4$ . We know  $32 \div 4 = 8$ ; 3200 is one hundred times bigger than 32, so the answer is one hundred times bigger:  $8 \times 100 = 800$ .

### Questions

1.  $9 \div 0.3$
3.  $0.8 \div 0.4$
5.  $3.4 \div 0.2$
7.  $6.4 \div 0.04$
9.  $36 \div 0.9$
11.  $35 \div 0.7$

2.  $7.2 \div 0.9$
4.  $3.6 \div 0.3$
6.  $7.2 \div 0.3$
8.  $65 \div 0.5$
10.  $27 \div 0.09$
12.  $2.8 \div 0.07$

### Hints / Notes

We use the times tables to find answers to simple divisions, for example if  $3 \times 9 = 27$  then  $27 \div 3 = 9$ .

*Trouble with this? Jump to: **Dividing Using the Times Tables or By Counting On** (p. 178)*

## 13.3 Multiplying Decimal Numbers with Two or Three Digits

There are two methods for multiplying decimal numbers with 2 or 3 digits. The first method is called the table method, the second method is called the lattice method.

### Multiplying using the Table Method

To multiply using the table method, we split up each number into tens, units, tenths and so on. We draw a grid to fit the numbers, writing the first number across the top of the table and the second number downwards, on the left-hand side of the table. We multiply the parts of each number, writing the results inside the table. To find the answer we add the results.

**Key point**

Draw a table. Split the numbers into tens, units, tenths and so on. Multiply each part, then add the results.

**Example 13.3**

$$3.6 \times 5.4 =$$

Split up the numbers:  $3.6 = 3 + 0.6$  and  $5.4 = 5 + 0.4$ . Draw the table and fill in the middle section by multiplying the parts ( $3 \times 5 = 15$ ,  $0.6 \times 5 = 3$ ,  $3 \times 0.4 = 1.2$ ,  $0.6 \times 0.4 = 0.24$ ).

$$\begin{array}{r} \times \quad 3 \quad 0.6 \\ 5 \quad 15 \quad 3 \\ 0.4 \quad 1.2 \quad 0.24 \end{array}$$

Add up the middle values:  $15 + 3 + 1.2 + 0.24 = 19.44$ .

$$3.6 \times 5.4 = 19.44$$

**Example 13.4**

$$8.3 \times 14.5 =$$

Split up the numbers:  $8.3 = 8 + 0.3$  and  $14.5 = 10 + 4 + 0.5$ . Draw the table then fill in the middle section.

$$\begin{array}{r} \times \quad 8 \quad 0.3 \\ 10 \quad 80 \quad 3 \\ 4 \quad 32 \quad 1.2 \\ 0.5 \quad 4 \quad 0.15 \end{array}$$

Add up the middle values:  $80 + 3 + 32 + 1.2 + 4 + 0.15 = 120.35$ .

$$14.5 \times 8.3 = 120.35$$

**Multiplying using the Lattice Method**

To multiply using the lattice method we start by drawing a grid. We draw one square across for each digit in the first number and one square downwards for each digit in the second number.

We add diagonal lines to the grid, drawing downwards and to the left from the top right corner of each square. We then write each number on the outside of the grid, one digit above each square and any decimal points lining up with the lines of the grid.

We multiply the digits, placing the tens above the diagonal and the units below the diagonal in each square.

We add the results diagonally, starting on the right, carrying any numbers into the diagonal column on the left. To place the new decimal point, we follow the line downwards from the decimal point

in the top number and follow the line across from the decimal point in the number on the right. We stop where the lines meet, then we trace along the diagonal from this point downwards. We place the decimal point in the answer here.

The answer is found from reading the outside digits from the top left to the bottom right of the grid.

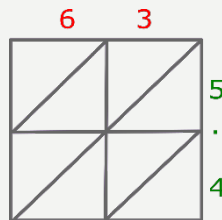
**Key point**

Draw a grid with diagonal lines. Write each digit around the edge of the grid then multiply the digits, tens above the diagonal, units below. Add the results diagonally, place the new decimal point along the diagonal from where the lines from the points meet.

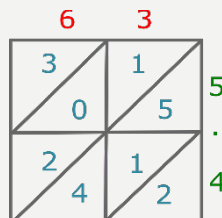
**Example 13.5**

$63 \times 5.4 =$

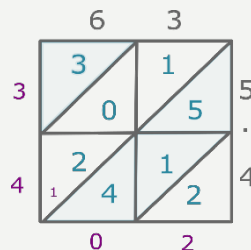
Draw a grid, then write the numbers along the outside of the grid.



$3 \times 5 = 15$ . Place 1 in the tens and 5 in the units.  $6 \times 5 = 30$ . Place 3 in the tens and 0 in the units.  $3 \times 4 = 12$ . Place 1 in the tens and 2 in the units.  $6 \times 4 = 24$ . Place 2 in the tens and 4 in the units.



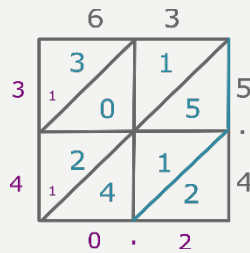
Add the numbers inside each *diagonal* column: write 2 beneath its diagonal column.  $5 + 1 + 4 = 10$ , write 0 beneath its column then write a small (carried) 1 in the next diagonal.  $2 + 0 + 1 + 1 = 4$ , write 4 at the end of this column. Write 3 at the end of its column.



Think of the top number, 63 as being 63.0. Follow the line vertically downwards from where the decimal point would be, until you reach the decimal point of the number on the right-hand

side of the grid, (between the 5 and the 4). From this point, trace along the diagonal line. Add the new decimal point for the answer here.

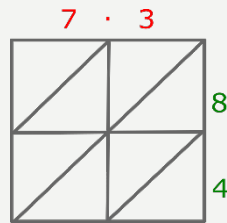
$$63 \times 5.4 = 340.2$$



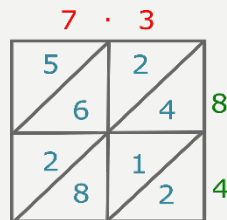
**Example 13.6**

$$7.3 \times 84 =$$

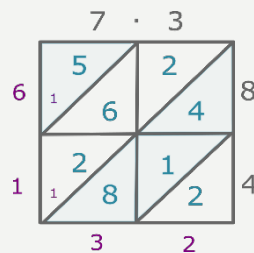
Draw a grid then write the numbers along the outside of the grid.



$3 \times 8 = 24$ . Place 2 in the tens and 4 in the units.  $7 \times 8 = 56$ . Place 5 in the tens and 6 in the units.  $3 \times 4 = 12$ . Place 1 in the tens and 2 in the units.  $7 \times 4 = 28$ . Place 2 in the tens and 8 in the units.

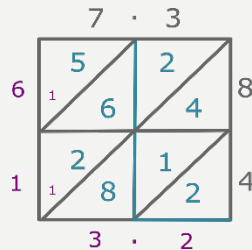


Add the numbers inside each *diagonal* column: write 2 beneath its diagonal column.  $4 + 1 + 8 = 13$ , write 3 beneath its column then write a small (carried) 1 in the next diagonal.  $2 + 6 + 2 + 1 = 11$ , write 1 at the end of this column then write a small (carried) 1 in the next diagonal.  $5 + 1 = 6$ , write 6 at the end of this column.



Follow the line vertically downwards from the decimal point in the top number (between the 7 and the 3). Think of the number 84, on the right-hand side of the grid, as being 84.0. Follow the line across horizontally from where the decimal point would be. Place the decimal point for the answer where the two lines meet, between the 3 and the 2.

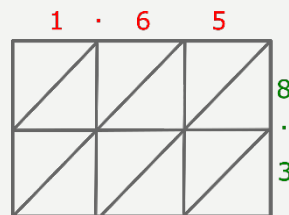
$$7.3 \times 84 = 613.2$$



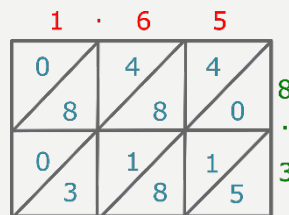
**Example 13.7**

$$1.65 \times 8.3 =$$

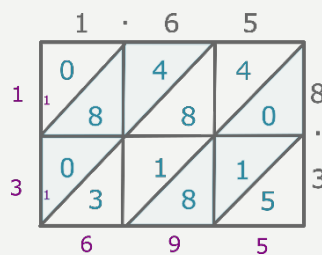
Draw a grid then write the numbers along the outside of the grid, placing the decimal points in their positions.



Fill in the first row of the grid by multiplying the top digits by 8. Fill in the second row by multiplying the top digits by 3.



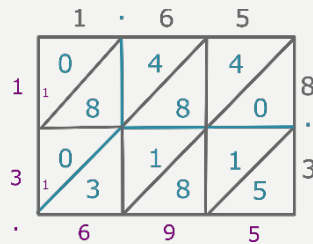
Add up along the *diagonal* columns, placing any carried numbers into the next grid column.



Follow the line downwards from the decimal point in the top number (between the 1 and the 6), at the same time follow the line horizontally across from the decimal point in the number

on the right (between the 8 and the 3). Trace along the diagonal line from the point where these lines meet. Place the decimal point for your answer here.

$$1.65 \times 8.3 = 13.695$$



### Questions — try both methods

1.  $18 \times 3.2$
3.  $5.2 \times 46$
5.  $4.75 \times 2.9$
7.  $65.5 \times 8.1$

2.  $26 \times 8.3$
4.  $1.2 \times 69$
6.  $3.94 \times 2.3$
8.  $7.8 \times 0.62$

#### Hints / Notes

Another way of placing the decimal point is to estimate the answer: for example,  $63 \times 5.4$  is roughly  $60 \times 5$ , so the answer is about 300.

*Trouble with this? Jump to: **Multiplying Decimal Numbers with 2 or 3 Digits** (p. 48)*

## 13.4 Dividing Whole Numbers and Decimals by Single Digit Numbers

To divide decimal numbers with two or three digits, we use the bus stop method shown below. As we divide, we place the decimal point in the answer above the decimal point in the question.

#### Key point

To divide decimal numbers, use the bus stop method. Place the decimal point above the decimal point in the question.

#### Example 13.8

$$4.98 \div 3 =$$

Write as a bus stop:

$$\begin{array}{r} 1.66 \\ 3 \overline{) 4.98} \end{array}$$

3 goes into 4 once with remainder 1. Place 1 above the 4 followed by a decimal point. Place 1 in front of the next digit; 9 becomes 19. 3 goes into 19 six times with remainder 1. Place 6 above the 19. Place 1 in front of the next digit; the 8 becomes 18. 3 goes into 18 six times. Place 6 above the 18.

$$4.98 \div 3 = 1.66$$

### Example 13.9

$$29.2 \div 4 =$$

Write as a bus stop:

$$\begin{array}{r} 07.3 \\ 4 \overline{) 29.2} \end{array}$$

4 goes into 2 zero times with remainder 2. Place 0 above the 2. Place 2 in front of the next digit; the 9 becomes 29. 4 goes into 29 seven times with 1 remainder. Place 7 above the 29 followed by a decimal point. Place 1 in front of the next digit. 4 goes into 12 three times. Place 3 above the 12.

$$29.2 \div 4 = 7.3$$

### Example 13.10

$$7.6 \div 5 =$$

Write as a bus stop:

$$\begin{array}{r} 1.52 \\ 5 \overline{) 7.60} \end{array}$$

5 goes into 7 once with remainder 2. Place 1 above the 7 followed by a decimal point. Place 2 in front of the next digit; the 6 becomes 26. 5 goes into 26 five times with 1 remainder. Place 5 above the 26. There is no next digit so add a zero to the number inside the bus stop. Place 1 in front of the new digit; the 0 becomes 10. 5 goes into 10 twice. Place 2 above the 10.

$$7.6 \div 5 = 1.52$$

## Questions A

Use the bus stop method to divide.

1.  $2 \overline{) 9.8}$

2.  $3 \overline{) 8.7}$

3.  $3 \overline{) 7.32}$

4.  $4 \overline{) 28.4}$

5. 
$$5 \overline{) 80.8}$$

7. 
$$6 \overline{) 7.74}$$

9. 
$$7 \overline{) 935.9}$$

11. 
$$8 \overline{) 7.384}$$

13. 
$$3 \overline{) 7.53}$$

15. 
$$3 \overline{) 8.145}$$

17. 
$$5 \overline{) 9.40}$$

19. 
$$5 \overline{) 7.4}$$

21. 
$$6 \overline{) 28.5}$$

6. 
$$5 \overline{) 3.35}$$

8. 
$$8 \overline{) 51.2}$$

10. 
$$9 \overline{) 0.963}$$

12. 
$$7 \overline{) 44.87}$$

14. 
$$4 \overline{) 6.48}$$

16. 
$$4 \overline{) 64.60}$$

18. 
$$4 \overline{) 9.94}$$

20. 
$$4 \overline{) 2.6}$$



If we are asked to divide *by* a decimal number, it is better to adjust the calculation first. Multiply both numbers by ten, until the number we are dividing by is a whole number, then we can divide normally.

**Example 13.11**

$$0.76 \div 0.5 =$$

Multiply both numbers by 10. The calculation becomes  $7.6 \div 5$ . Divide normally. Remember to add a zero to the number inside the bus stop, if you run out of digits.

$$\begin{array}{r} 1.52 \\ 5 \overline{) 7.60} \end{array}$$

$$0.76 \div 0.5 = 1.52$$

**Example 13.12**

$$53 \div 0.7 =$$

Multiply both numbers by 10. The calculation becomes  $530 \div 7$ . Divide normally. Remember to add zeroes to the number inside the bus stop, if you run out of digits.

$$\begin{array}{r} 75.714 \\ 7 \overline{) 530.0000} \end{array}$$

The calculation doesn't stop; we decide to stop after 3 decimal places. We write the answer to 2 decimal places.

$$53 \div 0.7 = 75.71 \text{ (2 d.p.)}$$

### Questions B

Adjust these calculations then use the bus stop method to divide.

1.  $2.66 \div 0.8$

3.  $5 \div 0.3$

5.  $4.346 \div 0.05$

7.  $3.154 \div 0.11$

2.  $23 \div 0.7$

4.  $0.97 \div 0.08$

6.  $0.6912 \div 0.003$

8.  $6.786 \div 0.12$

#### Hints / Notes

With some of these questions, you need to add a .0 at the end of the number. Multiplying both numbers by 10 does not change the result in a division!

*Trouble with this? Jump to: **Multiplying and Dividing with Decimal Numbers** (p. 47)*

## 13.5 Dividing Whole Numbers and Decimals by Two Digit Numbers

Division can be thought of as repeated subtraction. If we want to know how many times a number goes into something, we can keep subtracting the number until we can't take away any more. The amount of times we subtract is the answer.

For example, if we want to know how many times 5 goes into 21, we can repeatedly subtract 5 from 21:

$$21 - 5 = 16$$

Once

$$16 - 5 = 11$$

Twice

$$11 - 5 = 6$$

Three times

$$6 - 5 = 1$$

Four times — stop here as we only have 1 left.

Therefore 5 goes into 21, four times remainder one.

This principle is useful for dividing by two-digit numbers. We take away amounts (chunks) of the number until we cannot take away any more. We add up the amounts (chunks) that we have taken away to find the answer. This method is called the chunking method.

#### Key point

Take away chunks of the number then add up the chunks to find the answer.

#### Example 13.13

$$252 \div 14 =$$

$$\begin{array}{r}
 14 \overline{) 252} \\
 \underline{-140} \quad -10 \times 14 \\
 112 \\
 \underline{-70} \quad -5 \times 14 \\
 42 \\
 \underline{-42} \quad -3 \times 14 \\
 0
 \end{array}$$

Subtract 10 lots of 14.  $10 \times 14 = 140$ . There is 112 remaining. Subtract 5 lots of 14.  $5 \times 14 = 70$ . There is 42 remaining. Subtract 3 lots of 14.  $3 \times 14 = 42$ . Add the amounts of 14:  $10 + 5 + 3 = 18$ .

$$252 \div 14 = 18$$

**Example 13.14**

$$186 \div 15 =$$

$$\begin{array}{r}
 15 \overline{) 186} \\
 \underline{-150} \quad -10 \times 15 \\
 36 \\
 \underline{-30} \quad -2 \times 15 \\
 6 \\
 \underline{-6} \quad -0.4 \times 15 \\
 0
 \end{array}$$

Subtract 10 lots of 15.  $10 \times 15 = 150$ . There is 36 remaining. Subtract 2 lots of 15.  $2 \times 15 = 30$ . There is 6 remaining. ( $15 \times 4$  would be 60, but we have 6.) Subtract 0.4 lots of 15.  $0.4 \times 15 = 6$ . Add the amounts of 15:  $10 + 2 + 0.4 = 12.4$ .

$$186 \div 15 = 12.4$$

**Example 13.15**

$$633.6 \div 18 =$$

$$\begin{array}{r}
 18 \overline{) 633.6} \\
 \underline{-360} \quad -20 \times 18 \\
 273.6 \\
 \underline{-180} \quad -10 \times 18 \\
 93.6 \\
 \underline{-90} \quad -5 \times 18 \\
 3.6 \\
 \underline{-3.6} \quad -0.2 \times 18 \\
 0
 \end{array}$$

Subtract 20 lots of 18.  $18 \times 20 = 360$ . There is 273.6 remaining. Subtract 10 lots of 18.  $10 \times 18 = 180$ . There is 93.6 remaining. Subtract 5 lots of 18.  $5 \times 18 = 90$ . There is 3.6 remaining. ( $18 \times 2$  would be 36, but we have 3.6.) Subtract 0.2 lots of 18.  $0.2 \times 18 = 3.6$ . Add the amounts of 18:  $20 + 10 + 5 + 0.2 = 35.2$ .

$$633.6 \div 18 = 35.2$$

**Questions**

Use the chunking method to divide.

$$1. \quad 13 \overline{) 312}$$

$$3. \quad 16 \overline{) 496}$$

$$5. \quad 15 \overline{) 615}$$

$$7. \quad 13 \overline{) 468}$$

$$9. \quad 16 \overline{) 246.4}$$

$$11. \quad 13 \overline{) 254.8}$$

$$13. \quad 15 \overline{) 477}$$

$$15. \quad 13 \overline{) 704.6}$$

$$2. \quad 14 \overline{) 238}$$

$$4. \quad 14 \overline{) 322}$$

$$6. \quad 18 \overline{) 252}$$

$$8. \quad 14 \overline{) 588}$$

$$10. \quad 17 \overline{) 409.7}$$

$$12. \quad 14 \overline{) 2422}$$

$$14. \quad 18 \overline{) 292.5}$$

#### Hints / Notes

Taking away ten lots of a number all at once saves time.  $4 \times 15 = 60$ , therefore  $0.4 \times 15 = 6$ .

Trouble with this? Jump to: **Dividing Whole Numbers by Single Digits** (p. 178)

## 13.6 Problem Solving

Sometimes multiplication and division sums are presented as problems. We need to identify when to multiply and when to divide.

### Example 13.16

A bouncy ball weighs 3.5g. If there are 61 balls in a play pit, what do the balls weigh in total?  
1 ball = 3.5g, so 61 balls =  $3.5 \times 61$ g. The calculation is  $3.5 \times 61 =$ . Use the table method, the lattice method or a calculator:  $3.5 \times 61 = 213.5$ . The balls weigh 213.5g in total.

### Example 13.17

A parcel contains 18 silk scarves. If the contents of the parcel weigh 626.4g how much does a silk scarf weigh?

18 lots of something weigh 626.4g. The calculation is  $626.4 \div 18 =$ . Use the chunking method, the bus stop method or a calculator:  $626.4 \div 18 = 34.8$ . A silk scarf weighs 34.8g.

## Questions



1. A new car weighs 1820kg. A transporter moves 7 cars to a showroom. How much do the cars weigh in total?

2. There are 280 tomato seeds on a tray. Each tomato seed weighs approximately 2.1g, what do the contents of the tray weigh in total?

3. A box of chocolates weighs 201g. There are 15 chocolates in the box. How much does one chocolate weigh?

4. Jinny bought 12 cereal bars for £18.12. How much does one cereal bar cost?



5. Leo swam 7.5km each day for 14 days. How far did Leo swim in total?



6. The bamboo plant at the botanical gardens grew 45cm a day. How many days would it take for the bamboo plant to grow to 517.5cm?

## Chapter 14

# BIDMAS

### Grade 3

The letters in BIDMAS stand for Brackets, Indices, Division, Multiplication, Addition and Subtraction. This is the order in which we do calculations. If you try to calculate in a different order, you may get the wrong answer. For example, try to work out  $2 + 3 \times 5$  in your head, did you get 17 or 25? The answer, if you do the calculations in the correct order, is 17.

#### Key point

Order of calculations: Brackets, Indices, Divide, Multiply, Add then Subtract.

#### Example 14.1

$$3 + 4 \times 2 = 11$$

$$19 - 7 \times 2 = 5$$

$$15 + 6 \div 3 = 17$$

$$(4 + 5) \times 3 = 27$$

$$3 \times (6 - 2) = 12$$

$$(15 + 3) \div (9 - 6) = 6$$

$$3 = 6.$$

Multiply first:  $4 \times 2 = 8$ . Then add:  $3 + 8 = 11$ .

Multiply first:  $7 \times 2 = 14$ . Then subtract:  $19 - 14 = 5$ .

Divide first:  $6 \div 3 = 2$ . Then add:  $15 + 2 = 17$ .

Work out the brackets:  $4 + 5 = 9$ . Then multiply:  $9 \times 3 = 27$ .

Work out the brackets:  $6 - 2 = 4$ . Then multiply:  $3 \times 4 = 12$ .

Work out the brackets:  $15 + 3 = 18$  and  $9 - 6 = 3$ . Then divide:  $18 \div$

#### Note

What about Indices? They are shown in the next examples.

#### Questions A

1.  $2 + 4 \times 5$

3.  $20 - 8 \times 2$

5.  $12 + 6 \div 3$

7.  $4 + 3 \times 7$

9.  $17 - 5 \times 2$

11.  $10 + 15 \div 5$

13.  $9 + 2 \times 4$

15.  $16 - 3 \times 2$

17.  $4 + 8 \div 4$

19.  $6 + 8 \times 5$

21.  $35 - 10 \times 2$

23.  $22 + 12 \div 4$

2.  $(17 - 7) \times 3$

4.  $4 \times (9 - 3)$

6.  $(5 + 4) \div 3$

8.  $(11 + 5) \times 2$

10.  $7 \times (5 - 2)$

12.  $(3 + 3) \times (2 + 3)$

14.  $5 \times (12 - 5)$

16.  $10 - (9 - 3)$

18.  $(16 + 4) \div (2 + 3)$

20.  $(18 + 5) \times 2$

22.  $29 - (15 - 5)$

24.  $(12 + 3) \div (2 + 3)$

The following examples have numbers raised to a power, or in other words, indices.

**Example 14.2**

$3^2 + 21 \div 3 = 16$     Work out the indices:  $3^2 = 9$ . Then divide:  $21 \div 3 = 7$ . Finally, add:  $9 + 7 = 16$ .

$(4 - 2)^3 \times 5 = 40$     Work out the brackets:  $4 - 2 = 2$ . Then the indices:  $2^3 = 8$ . Finally, multiply:  $8 \times 5 = 40$ .

**Questions B**

1.  $5^2 - 6 \times 2$
3.  $8^2 - 5 \times 2$
5.  $(7 + 3)^2 - 5 \times 4$
7.  $(2 + 5)^2 + 3 \times 2$
9.  $18 - (4 - 2)^3$
11.  $10 + 3^2 \times 5$
13.  $100 - 8 \times 3^2$
15.  $24 + 6 \div 3$
2.  $(18 - 7) \times 3^2$
4.  $2^2 \times (9 - 3)$
6.  $(14 - 4)^2 \div (2 + 3)$
8.  $1 + 3 \times 2^3$
10.  $8 + 6 \times 2^2$
12.  $(2 + 5)^2 - 5 \times 2$
14.  $39 - (5 - 2)^3$
16.  $(7 - 4)^2 \times (12 - 5)$

**Hints / Notes**

Work out any brackets first, then work out indices or powers. Multiplying or dividing is next. Save any adding or subtracting until last!

*Trouble with this? Jump to: **Square Numbers and Square Roots** (p. 20); **Cube Numbers and Cube Roots** (p. 22)*

## Chapter 15

# Inverse Operations

### Grade 2

An operation is something we do to a number. We can add, subtract, multiply or divide a number, these are called operations. We can also raise numbers to powers, such as squaring or cubing numbers, these are also operations that we can do to a number.

An inverse operation undoes the operation. The opposite of adding is subtracting, the opposite of multiplying is dividing. If we add numbers, for example,  $3 + 4 = 7$ , the inverse operations are  $7 - 4 = 3$  and  $7 - 3 = 4$ . If we multiply numbers, for example,  $3 \times 4 = 12$ , the inverse operations are  $12 \div 4 = 3$  and  $12 \div 3 = 4$ .

### Key point

An inverse operation does the opposite of an operation.

### Example 15.1

$$5 + 7 = 12 \quad 12 - 7 = 5 \text{ and } 12 - 5 = 7$$

$$5 \times 3 = 15 \quad 15 \div 3 = 5 \text{ and } 15 \div 5 = 3$$

### Questions A

Write the inverse operations for the following calculations.

1.  $5 + 8 = 13$

3.  $4 + 6 = 10$

5.  $7 + 8 = 15$

7.  $9 + 2 = 11$

2.  $6 + 2 = 8$

4.  $12 + 5 = 17$

6.  $18 + 4 = 22$

8.  $13 + 3 = 16$

### Questions B

Write the inverse operations for the following calculations.

1.  $4 \times 6 = 24$

3.  $7 \times 6 = 42$

5.  $5 \times 8 = 40$

7.  $9 \times 5 = 45$

2.  $12 \times 3 = 36$

4.  $8 \times 9 = 72$

6.  $7 \times 3 = 21$

8.  $11 \times 7 = 77$

### Questions C

Which of these calculations is correct? Check by writing an inverse operation.

1.  $40 \div 5 = 6$

3.  $72 \div 8 = 9$

2.  $63 \div 9 = 8$

4.  $72 \div 6 = 12$

$$\begin{aligned} 5. 56 \div 6 &= 7 \\ 7. 93 \div 3 &= 31 \end{aligned}$$

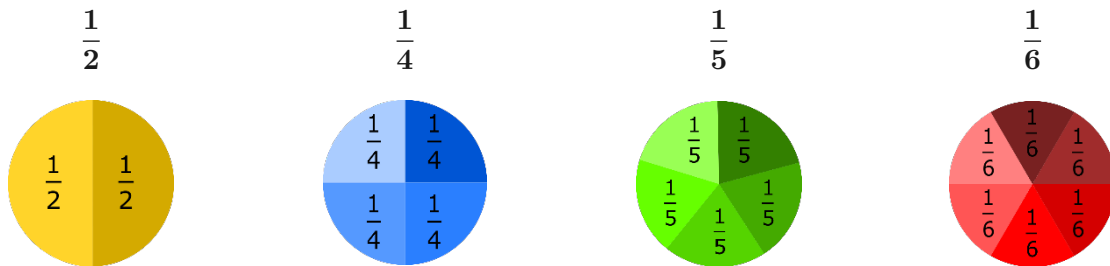
$$\begin{aligned} 6. 25 \div 4 &= 5 \\ 8. 48 \div 8 &= 6 \end{aligned}$$

## Chapter 16

# Fractions, Decimals and Percentages

### 16.1 Fractions of a Whole

**Grade 2** The table below shows a whole circle split into different fractions. Halves are bigger than quarters, quarters are bigger than sixths and so on.



There are 2 Halves in a Whole.    There are 4 Quarters in a Whole.    There are 5 Fifths in a Whole.    There are 6 Sixths in a Whole.

Fractions are like decimal numbers, they are part of a whole one. Fractions are written with the numerator on top and the denominator on the bottom.

The denominator tells you how many make a whole one. The numerator tells you how many you have. If you have  $\frac{3}{10}$  of a bar of chocolate then out of the 10 pieces that make up the bar, you have 3 pieces.

$\frac{3}{10}$  also means 3 parts out of every 10, so if your bar had 20 pieces, you would have 6 pieces.

#### Key point

Fractions are written with one number (the numerator) on top and one number (the denominator) on the bottom.

#### Example 16.1

$\frac{3}{4}$  is a whole one. Add  $\frac{1}{4}$ .  
 $\frac{2}{6}$  is a whole one. Add  $\frac{4}{6}$ .  
 $\frac{2}{9}$  is a whole one. Add  $\frac{7}{9}$ .

#### Questions A

For the fractional quantities below, how much should be added to make a whole one?

- $\frac{1}{8}$
- $\frac{3}{5}$
- $\frac{3}{10}$
- $\frac{5}{12}$
- $\frac{1}{9}$
- $\frac{1}{4}$

- 7.  $\frac{6}{7}$
- 9.  $\frac{9}{10}$
- 11.  $\frac{8}{11}$

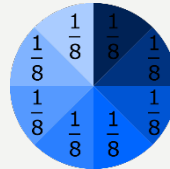
- 8.  $\frac{1}{2}$
- 10.  $\frac{2}{3}$
- 12.  $\frac{1}{3}$

Fractions are used to solve real life problems.

**Example 16.2**

Donna eats  $\frac{5}{8}$  of a pizza. What fraction of the pizza is left?

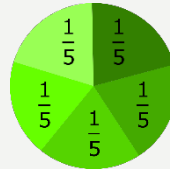
A whole pizza is  $\frac{8}{8}$ .



If Donna eats  $\frac{5}{8}$  there are  $\frac{3}{8}$  left.

Jake has picture cards. Only  $\frac{2}{5}$  of his cards have plastic wallets. What fraction of his cards don't have plastic wallets?

All of Jake's cards would be  $\frac{5}{5}$ .



If  $\frac{2}{5}$  have plastic wallets, then there are  $\frac{3}{5}$  left.  
 $\frac{3}{5}$  of Jake's cards do not have plastic wallets.

**Questions B**

1. Tina picks some flowers from the garden. She picks  $\frac{1}{4}$  of the flowers. What fraction are left in the garden?



2. Jake likes to watch videos.  $\frac{1}{3}$  of the videos are about maths. What fraction are not about maths?
3. Henry collects toy cars,  $\frac{3}{5}$  of them are sports cars. What fraction are not sports cars?
4. Stefan likes to run along the beach. If he runs  $\frac{1}{5}$  of the way along the beach, what fraction does he have left to run?



5. Guy likes photography, he photographs animals and birds.  $\frac{3}{10}$  of his photographs are of birds. What fraction are photographs of animals?
6. Nadine likes climbing. She climbed  $\frac{7}{9}$  of the way up a tall peak. What fraction does she have left to climb?

## 16.2 Fractions of Shapes

**Grade 2** To find the fraction of a shape that has been shaded, count the total number of parts and write this number in the denominator (on the bottom). Count the number of shaded parts and write this number in the numerator (on the top).

### Key point

Write the number of shaded parts in the numerator and the total number of parts in the denominator.

### Example 16.3



1 out of 4 squares have been shaded.

The fraction is  $\frac{1}{4}$ .



5 out of 6 squares have been shaded.

The fraction is  $\frac{5}{6}$ .

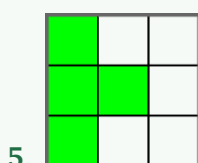
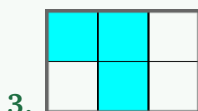


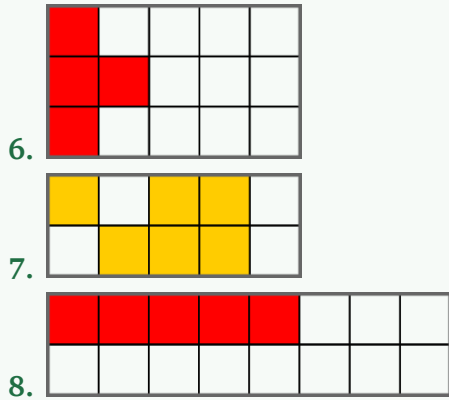
3 out of 8 squares have been shaded.

The fraction is  $\frac{3}{8}$ .

### Questions A

What fraction of the following shapes have been shaded?



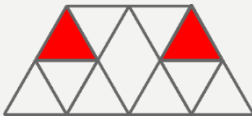


**Hints / Notes**  
Check that the parts are the same size first.

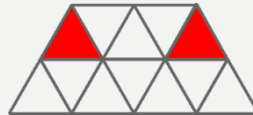
*Trouble with this? Jump to:* **Fraction Names** (p. 189); **Simplifying Fractions by Cancelling Down** (p. 74)

**Key point**  
If a shape contains unequal parts, first add lines to make the parts equal.

**Example 16.4**

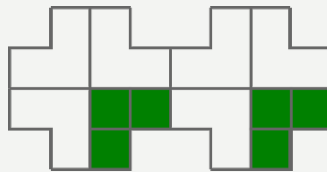
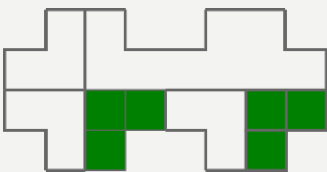


Add a line to the middle diamond, to make all the parts equal.



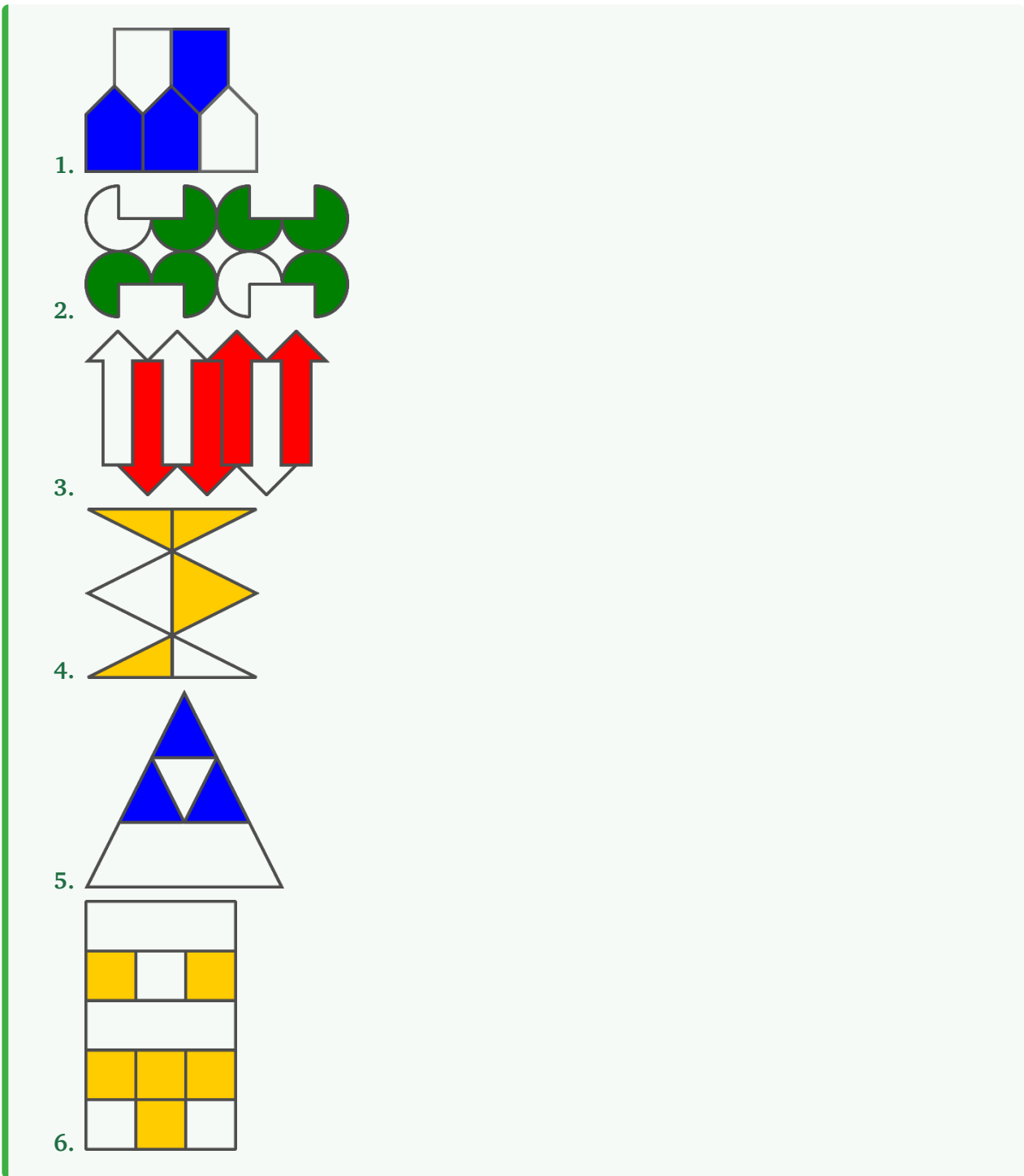
There are 12 triangles and 2 of them are shaded, the fraction is  $\frac{2}{12}$  or  $\frac{1}{6}$

Add two lines to make the parts equal.



There are 8 shapes and 2 are shaded in. The fraction is  $\frac{2}{8}$  or  $\frac{1}{4}$

**Questions B**  
What fraction of the following shapes have been shaded?



### 16.3 Fractions of a Quantity

**Grade 3** To find a simple fraction of a quantity we divide by the denominator (bottom). For example, to find one tenth of a quantity we divide by 10, to find one half of a quantity we divide by 2. To find one fifth of a quantity we divide by 5 and so on.

**Key point**  
To find a simple fraction of a quantity we divide by the denominator.

**Example 16.5**

$$\begin{array}{ll} \frac{1}{10} \text{ of } 30 & 30 \div 10 = 3 \\ \frac{1}{4} \text{ of } 40 & 40 \div 4 = 10 \\ \frac{1}{6} \text{ of } 66 & 66 \div 6 = 11 \end{array}$$

**Questions A**

Find these quantities.

- |                         |                           |
|-------------------------|---------------------------|
| 1. $\frac{1}{10}$ of 50 | 2. $\frac{1}{3}$ of 18    |
| 3. $\frac{1}{4}$ of 36  | 4. $\frac{1}{6}$ of 48    |
| 5. $\frac{1}{4}$ of 25  | 6. $\frac{1}{9}$ of 45    |
| 7. $\frac{1}{7}$ of 28  | 8. $\frac{1}{2}$ of 36    |
| 9. $\frac{1}{8}$ of 24  | 10. $\frac{12}{11}$ of 77 |

**Hints / Notes**

Divide using a calculator or by using the bus stop method, if you can't remember how then jump to dividing! These simple fractions have a numerator of 1.

*Trouble with this? Jump to: **Dividing Using the Times Tables or By Counting On** (p. 178)*

To find a fraction of a quantity we divide by the denominator (bottom). If the numerator (top) is not equal to one, then we also multiply by the numerator. For example, to find three tenths of a quantity, we find one tenth, then multiply by three. To find four fifths of a quantity, we find one fifth, then multiply by four and so on.

**Key point**

To find a fraction of a quantity we divide by the denominator and multiply by the numerator.

**Example 16.6**

$$\frac{3}{10} \text{ of } 40$$

Find  $\frac{1}{10}$  of 40:

$$40 \div 10 = 4$$

Multiply by 3:

$$4 \times 3 = 12$$

$$\frac{3}{10} \text{ of } 40 = 12$$

$$\frac{2}{5} \text{ of } 35$$

Find  $\frac{1}{5}$  of 35:

$$35 \div 5 = 7$$

Multiply by 2:

$$7 \times 2 = 14$$

$$\frac{2}{5} \text{ of } 35 = 14$$

## Questions B

Find these quantities.

1.  $\frac{2}{3}$  of 15
3.  $\frac{3}{4}$  of 36
5.  $\frac{1}{3}$  of 55
7.  $\frac{5}{9}$  of 36

2.  $\frac{7}{10}$  of 90
4.  $\frac{3}{10}$  of 24
6.  $\frac{1}{2}$  of 42
8.  $\frac{1}{6}$  of 30

### Hints / Notes

You can multiply by the top first, then divide the bottom after, if you prefer.

*Trouble with this? Jump to: **Multiplying Using the Times Tables or By Counting On** (p. 173)*

Fractions are used to solve real life problems. Write the problem as a calculation, before working it out.

### Example 16.7

Sahid has 30 books.  $\frac{1}{3}$  of his books are science fiction. How many are science fiction?

The calculation is:

$$\frac{1}{3} \text{ of } 30 =$$

Divide by 3.

$$30 \div 3 = 10$$

There are ten science fiction books.

Jessica has 15 dancing lessons in a term.  $\frac{3}{5}$  of them are ballet lessons. How many are ballet lessons?

The calculation is:

$$\frac{3}{5} \text{ of } 15 =$$

Find one fifth:

$$\frac{1}{5} \text{ of } 15 = 3$$

Multiply by 3 to find  $\frac{3}{5}$ .

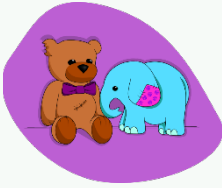
$$3 \times 3 = 9$$

Nine of the lessons are ballet lessons.

## Questions C

1. Theo had 60 text messages. Theo read  $\frac{1}{5}$  of the messages. How many messages did Theo read?
2. Eleanor grows cacti plants. Eleanor's tallest cacti is 90cm high. Eleanor's smallest cacti is  $\frac{3}{10}$  of this size. How tall is Eleanor's smallest plant?
3. Zoe earns £16 per week from her weekend job. Zoe spends  $\frac{3}{8}$  of her money on sweets and music. How much does Zoe spend on sweets and music?

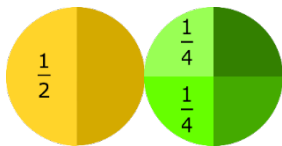
4. Lewis collects soft toys, he has a collection of 40.  $\frac{2}{5}$  of Lewis' soft toys are elephants. How many are elephants?



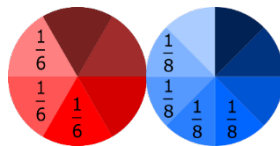
5. Harry wanted to cycle 24 miles to the forest. Harry got a flat tyre after travelling  $\frac{3}{8}$  of the way. How far did Harry get before his tyre went flat?
6.  $\frac{1}{5}$  of 65 million people in Great Britain are ex-smokers. How many are ex-smokers?

## 16.4 Equivalent Fractions

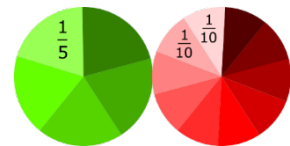
**Grade 2** Equivalent fractions are fractions that are the same, they represent the same part of a whole. For example, we could eat half of a cake, or two quarters of a cake, this is the same amount of cake! One half is the same as two quarters, it is the same as three sixths and so on.



$$\frac{1}{2} = \frac{2}{4}$$



$$\frac{3}{6} = \frac{4}{8}$$



$$\frac{1}{5} = \frac{2}{10}$$

We do not have to draw diagrams to find equivalent fractions. If the numerator (top) and denominator (bottom) of a fraction are multiplied by the same number, then we have made an equivalent fraction. For example, if we multiply the top and bottom of  $\frac{1}{2}$  by 2, we have  $\frac{2}{4}$ .

### Key point

Make an equivalent fraction by multiplying the numerator and denominator by the same number.

### Example 16.8

The strip below has been split into different fractions,  $\frac{6}{12}$  is the same as  $\frac{5}{10}$ . Also  $\frac{4}{6}$  is the same as  $\frac{8}{12}$ .

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

### Questions A

Complete the table by multiplying the fraction in the left column.

1.

	$\times 2$	$\times 3$	$\times 4$	$\times 5$
$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$
$\frac{1}{4}$				
$\frac{1}{3}$				
$\frac{1}{5}$				
$\frac{1}{10}$				
$\frac{3}{4}$				

### Questions B

1. Use the table below to find the missing equivalent fractions.

$$\frac{1}{3} = \frac{2}{6} = \frac{\square}{9} = \frac{\square}{12} = \frac{\square}{15}$$

$$\frac{2}{3} = \frac{\square}{6} = \frac{3}{9} = \frac{\square}{12} = \frac{\square}{15}$$

$$\frac{6}{9} = \frac{\square}{12} = \frac{\square}{15}$$

$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	
$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$	
	$\frac{1}{6}$			$\frac{1}{6}$			
		$\frac{1}{3}$					

2. Complete the table below then find the missing numbers for these fractions.

$$\frac{3}{4} = \frac{6}{\square} = \frac{\square}{12} = \frac{\square}{20}$$

$$\frac{5}{8} = \frac{\square}{16}$$

		$\frac{1}{20}$	
$\frac{1}{16}$			
			$\frac{1}{8}$
	$\frac{1}{4}$		

3. Find the missing equivalent fractions by multiplying the first fraction.

$$\frac{1}{5} = \frac{2}{\square} = \frac{\square}{15} = \frac{4}{\square} = \frac{\square}{30}$$

To find missing numbers in equivalent fractions, we work out how the fraction has changed, then we know the missing number.

**Example 16.9**

Find the missing numbers for these equivalent fractions.

$$\frac{1}{5} = \frac{3}{\square}$$

$$1 \times 3 = 3$$

The numerator has been multiplied by 3.

Multiply the denominator by 3:

$$5 \times 3 = 15$$

$$\frac{1}{5} = \frac{3}{15}$$

$$\frac{1}{3} = \frac{\square}{12}$$

$$3 \times 4 = 12$$

The denominator has been multiplied by 4.

Multiply the numerator by 4:

$$1 \times 4 = 4$$

$$\frac{5}{\square} = \frac{20}{28}$$

$$\frac{1}{3} = \frac{4}{12}$$

$$5 \times 4 = 20$$

The numerator has been multiplied by 4.

Working backwards, we divide the denominator by 4.

$$28 \div 4 = 7$$

$$\frac{5}{7} = \frac{20}{28}$$

## Questions C

Find the missing numbers for these equivalent fractions.

1.  $\frac{1}{\square} = \frac{3}{12}$
3.  $\frac{3}{\square} = \frac{12}{24}$
5.  $\frac{5}{\square} = \frac{20}{40}$
7.  $\frac{7}{\square} = \frac{28}{56}$
9.  $\frac{9}{\square} = \frac{36}{72}$
11.  $\frac{11}{\square} = \frac{44}{88}$
13.  $\frac{13}{\square} = \frac{52}{104}$
15.  $\frac{15}{\square} = \frac{60}{120}$
17.  $\frac{17}{\square} = \frac{68}{136}$
19.  $\frac{19}{\square} = \frac{76}{152}$
21.  $\frac{21}{\square} = \frac{84}{168}$
23.  $\frac{23}{\square} = \frac{92}{460}$

2.  $\frac{1}{3} = \frac{4}{\square}$
4.  $\frac{3}{5} = \frac{12}{\square}$
6.  $\frac{2}{3} = \frac{8}{\square}$
8.  $\frac{4}{7} = \frac{24}{\square}$
10.  $\frac{5}{8} = \frac{9}{\square}$
12.  $\frac{3}{5} = \frac{18}{\square}$
14.  $\frac{2}{3} = \frac{18}{\square}$
16.  $\frac{4}{7} = \frac{20}{\square}$
18.  $\frac{5}{8} = \frac{24}{\square}$
20.  $\frac{3}{5} = \frac{24}{\square}$
22.  $\frac{4}{5} = \frac{36}{\square}$
24.  $\frac{5}{22} = \frac{80}{\square}$

## 16.5 Simplifying Fractions by Cancelling Down

**Grade 2** Equivalent fractions are found by cancelling down or simplifying. We find a value that divides into the numerator (top) and the denominator (bottom), this is a factor. We divide by the factor.

### Key point

To cancel down a fraction, divide the numerator and the denominator by the same number.

### Example 16.10

Cancel down the following fractions.

$$\frac{6}{10}$$

Both numbers are even, therefore 2 is a factor.

Divide the numerator and denominator by 2.

$$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

$$\frac{6}{10} = \frac{3}{5}$$

Both numbers can be divided by 3, therefore 3 is a factor.

Divide the numerator and denominator by 3.

$$\frac{9}{15}$$

$$\frac{15}{40}$$

$$\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$$

$$\frac{9}{15} = \frac{3}{5}$$

Both numbers can be divided by 5, therefore 5 is a factor.

Divide the numerator and denominator by 5.

$$\frac{15 \div 5}{40 \div 5} = \frac{3}{8}$$

$$\frac{15}{40} = \frac{3}{8}$$

## Questions A

Cancel down these fractions.

1.  $\frac{2}{65}$
3.  $\frac{30}{42}$
5.  $\frac{36}{48}$
7.  $\frac{40}{45}$
9.  $\frac{45}{50}$
11.  $\frac{50}{100}$
13.  $\frac{16}{18}$
15.  $\frac{16}{25}$
17.  $\frac{48}{56}$
19.  $\frac{69}{75}$
21.  $\frac{51}{59}$
23.  $\frac{50}{116}$

2.  $\frac{8}{18}$
4.  $\frac{34}{35}$
6.  $\frac{70}{75}$
8.  $\frac{21}{27}$
10.  $\frac{38}{38}$
12.  $\frac{54}{54}$
14.  $\frac{80}{80}$
16.  $\frac{50}{50}$
18.  $\frac{72}{72}$
20.  $\frac{72}{72}$
22.  $\frac{80}{80}$
24.  $\frac{54}{75}$

### Hints / Notes

If you can't find a factor, check if one of the numbers divides into the other number. For example,  $\frac{25}{125}$ , 25 goes into 125!

Trouble with this? Jump to: **Factors of a Number** (p. 39)

## Questions B

Which of the fractions on the top row are equivalent to the fractions in the left column?

- 1.

	$\frac{54}{72}$	$\frac{5}{20}$	$\frac{28}{56}$	$\frac{8}{12}$	$\frac{10}{30}$	$\frac{9}{18}$	$\frac{12}{60}$	$\frac{30}{48}$
$\frac{1}{2}$								
$\frac{1}{4}$								
$\frac{1}{3}$								
$\frac{3}{4}$								
$\frac{1}{5}$								

**Hints / Notes**  
 Cancel down the fractions in the top row to find equivalent fractions.

*Trouble with this? Jump to: **Factors of a Number** (p. 39)*

## 16.6 Changing Mixed Numbers to Improper Fractions

**Grade 2** A mixed number is a fraction that has a whole number part. To change a mixed number to an improper fraction, the whole number part is changed into a fraction with the same denominator. The two fractions are then added. Two methods for changing mixed numbers are shown below.

### Method 1

The mixed number has a whole number part and a fraction part. Change the whole number part into a fraction with the same denominator (bottom), then add the fractions.

**Key point**  
 To change to an improper fraction, change the whole part into a fraction, then add the fractions.

**Example 16.11**

$1\frac{3}{4}$

$2\frac{1}{10}$

Change the whole number into quarters to match the fraction. 1 whole is 4 quarters.

Add 3 quarters.

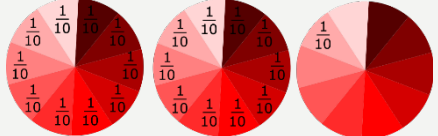
$$\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

(4 quarters plus 3 quarters equals 7 quarters)

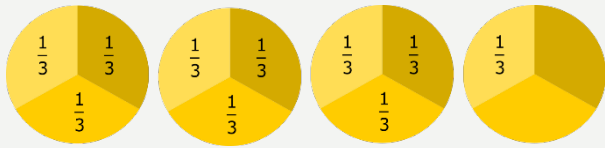
$$1\frac{3}{4} = \frac{7}{4}$$

Change the whole number into tenths.

1 whole is 10 tenths. 2 wholes are 20 tenths.



3  $\frac{1}{3}$



Add 1 tenth.  
 $\frac{20}{10} + \frac{1}{10} = \frac{21}{10}$   
 (20 tenths plus 1 tenth equals 21 tenths)  
 $2\frac{1}{10} = \frac{21}{10}$   
 Change the whole number thirds.  
 1 whole is 3 thirds. 3 wholes are 9 thirds.  
 Add 1 third.  
 $\frac{9}{3} + \frac{1}{3} = \frac{10}{3}$   
 $3\frac{1}{3} = \frac{10}{3}$

**Method 2**

A mixed number can be changed to an improper fraction by multiplying the whole number part by the denominator (bottom) and then adding the numerator (top).

**Key point**  
 To change to an improper fraction, multiply the whole part by the denominator then add the numerator. Write over the denominator.

**Example 16.12**

$1\frac{3}{4}$	$4 \times 1 = 4$
	$4 + 3 = 7$
	$1\frac{3}{4} = \frac{7}{4}$
$2\frac{1}{10}$	$10 \times 2 = 20$
	$20 + 1 = 21$
	$2\frac{1}{10} = \frac{21}{10}$
$3\frac{1}{3}$	$3 \times 3 = 9$
	$9 + 1 = 10$
	$3\frac{1}{3} = \frac{10}{3}$

**Questions**  
 Change these to improper fractions, try both methods.

1. $1\frac{1}{5}$	2. $1\frac{1}{3}$
3. $2\frac{1}{4}$	4. $2\frac{3}{4}$
5. $3\frac{1}{7}$	6. $2\frac{4}{5}$
7. $3\frac{1}{2}$	8. $1\frac{1}{10}$
9. $7\frac{2}{3}$	10. $2\frac{3}{10}$
11. $8\frac{1}{12}$	12. $5\frac{3}{4}$

**Hints / Notes**

Mixed numbers are whole numbers with fractions. Improper fractions are top heavy – the numerator is bigger than the denominator. Numerator – Top, Denominator – Bottom.

*Trouble with this? Jump to: **Fractions of a Whole** (p. 64)*

## 16.7 Changing Improper Fractions to Mixed Numbers

**Grade 2** An improper fraction is top heavy, the numerator (top) is larger than the denominator (bottom). To change an improper fraction to a mixed number, divide the numerator by the denominator. Write the remainder as a fraction, using the original denominator.

**Key point**

To change to a mixed number, divide the numerator by the denominator. Write the result, with the remainder over the denominator.

**Example 16.13**

Change these improper fractions to mixed numbers.

$$\frac{9}{4}$$

$$9 \div 4 = 2 \text{ remainder } 1$$

$$\frac{9}{4} = 2\frac{1}{4}$$

$$\frac{23}{5}$$

$$23 \div 5 = 4 \text{ remainder } 3$$

$$\frac{23}{5} = 4\frac{3}{5}$$

$$\frac{10}{3}$$

$$10 \div 3 = 3 \text{ remainder } 1$$

$$\frac{10}{3} = 3\frac{1}{3}$$

**Questions**

Change these improper fractions to mixed numbers.

1.  $\frac{17}{4}$
3.  $\frac{34}{7}$
5.  $\frac{10}{5}$
7.  $\frac{19}{25}$
9.  $\frac{28}{11}$
11.  $\frac{23}{11}$

2.  $\frac{13}{17}$
4.  $\frac{17}{6}$
6.  $\frac{5}{7}$
8.  $\frac{22}{41}$
10.  $\frac{41}{45}$
12.  $\frac{45}{7}$

**Hints / Notes**

Mixed numbers are whole numbers with fractions. Numerator – The top of the frac-

*Trouble with this? Jump to: **Dividing Whole Numbers by Single Digits** (p. 178)*

tion. Denominator – The bottom of the fraction.

## 16.8 Ordering Fractions by Comparing to One Half or One Whole

**Grade 3** To put fractions in order, smallest to largest, we check their denominators. If the fractions have equal denominators, then we can order the fractions by comparing their numerators. For example, if we have  $\frac{6}{10}$  and  $\frac{3}{10}$ , 6 is larger than 3, therefore  $\frac{6}{10}$  is larger than  $\frac{3}{10}$ .

If the fractions have different denominators, then we can compare each fraction to one half or one whole.



From the diagram, we can see that  $\frac{6}{10}$  is larger than  $\frac{3}{8}$ . What happens if we don't have a diagram? We can, instead think about the tenths and the eighths. For example,  $\frac{5}{10}$  is one half and  $\frac{4}{8}$  is one half. The fraction  $\frac{6}{10}$  is bigger than  $\frac{5}{10}$  therefore  $\frac{6}{10}$  is *more* than one half. The fraction  $\frac{3}{8}$  is less than  $\frac{4}{8}$ , therefore  $\frac{3}{8}$  is *less* than one half. Therefore,  $\frac{6}{10}$  is larger than  $\frac{3}{8}$ . The order is  $\frac{3}{8}, \frac{6}{10}$ .

### Key point

To order fractions, compare each fraction to one half or one whole.

### Example 16.14

Which of the two fractions is the largest?

$\frac{5}{8}$  and  $\frac{4}{10}$

We know that  $\frac{4}{8}$  is one half, therefore  $\frac{5}{8}$  is more than one half.

We know that  $\frac{5}{10}$  is one half, therefore  $\frac{4}{10}$  is less than one half.

$\frac{5}{8}$  is larger than  $\frac{4}{10}$ .

$\frac{8}{9}$  and  $\frac{5}{8}$

We know that  $\frac{9}{9}$  is one whole, therefore  $\frac{8}{9}$  is almost one whole.

We know that  $\frac{8}{8}$  is one whole, therefore  $\frac{5}{8}$  is much less than one whole.

$\frac{8}{9}$  is larger than  $\frac{5}{8}$ .

## Questions

Which of the two fractions is the largest?

1.  $\frac{3}{3}$  and  $\frac{7}{2}$
3.  $\frac{1}{2}$  and  $\frac{3}{2}$
5.  $\frac{4}{7}$  and  $\frac{4}{7}$
7.  $\frac{7}{10}$  and  $\frac{3}{11}$

2.  $\frac{3}{4}$  and  $\frac{3}{10}$
4.  $\frac{1}{4}$  and  $\frac{3}{10}$
6.  $\frac{3}{5}$  and  $\frac{3}{10}$
8.  $\frac{3}{6}$  and  $\frac{3}{8}$

### Hints / Notes

Think, is the fraction more than a half or less than a half?

*Trouble with this? Jump to: **Ordering Fractions with the Same Denominator** (p. 191)*

## 16.9 Ordering Fractions by Using Equivalent Fractions

**Grade 3** To put fractions in order, smallest to largest, we check their denominators. If the fractions have equal denominators, then we can order the fractions by comparing their numerators.

If fractions have different denominators, we can use equivalent fractions to make the denominators (bottom) the same, then we can compare their numerators (top).

### Key point

Make the denominators equal, then compare their numerators.

### Example 16.15

Put these fractions in order, smallest to largest.

$$\frac{4}{9}, \frac{5}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{1}{9}$$

Change the fractions with the smaller denominators,  $\frac{1}{3}$  and  $\frac{2}{3}$ , to match the larger ones.

$$\frac{1 \times 3}{3 \times 3} = \frac{3}{9}, \quad \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

The fractions become:

$$\frac{4}{9}, \frac{5}{9}, \frac{3}{9}, \frac{6}{9}, \frac{7}{9}, \frac{1}{9}$$

Put the fractions in order, smallest to largest.

$$\frac{1}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}$$

Write the fractions in their original form.

The order is:  $\frac{1}{9}, \frac{1}{3}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{7}{9}$ .

## Questions

Put these fractions in order, smallest to largest.

1.  $\frac{4}{10}, \frac{3}{5}, \frac{1}{10}, \frac{7}{10}, \frac{1}{5}, \frac{9}{10}$
2.  $\frac{1}{8}, \frac{3}{4}, \frac{5}{8}, \frac{1}{2}, \frac{7}{8}, \frac{1}{4}$

3.  $\frac{3}{20}, \frac{4}{5}, \frac{1}{10}, \frac{7}{10}, \frac{2}{5}, \frac{9}{20}$
4.  $\frac{3}{8}, \frac{5}{16}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, \frac{1}{16}$
5.  $\frac{4}{9}, \frac{2}{3}, \frac{1}{9}, \frac{7}{27}, \frac{5}{9}, \frac{11}{27}$
6.  $\frac{5}{12}, \frac{8}{24}, \frac{7}{8}, \frac{5}{6}, \frac{1}{8}, \frac{11}{12}$

**Hints / Notes**

First, change the fractions, so that they all have the same denominator. Remember to write the fractions in the answer in their original form.

*Trouble with this? Jump to: **Ordering Fractions with the same denominator** (p. 191); **Equivalent Fractions** (p. 71)*

## 16.10 Adding and Subtracting Fractions

**Grade 3** Before adding or subtracting fractions, we check to see if their denominators are equal. If the denominators (bottom) are equal, then we add or subtract the numerators (top). The answer is written above the original denominator.

**Key point**

If the denominators are equal, add or subtract the numerators.

**Example 16.16**

$$\frac{1}{9} + \frac{4}{9} =$$

$$1 + 4 = 5$$

$$\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$$

$$\frac{6}{7} - \frac{2}{7} =$$

$$6 - 2 = 4$$

$$\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$$

**Questions A**

1.  $\frac{3}{5} + \frac{4}{5} =$
3.  $\frac{5}{6} - \frac{1}{6} =$
5.  $\frac{7}{8} + \frac{1}{8} =$
7.  $\frac{3}{5} - \frac{1}{5} =$

2.  $\frac{3}{9} + \frac{2}{9} =$
4.  $\frac{2}{6} - \frac{3}{6} =$
6.  $\frac{7}{15} + \frac{4}{15} =$
8.  $\frac{7}{11} - \frac{4}{11} =$

**Hints / Notes**

Add or subtract the tops and put over the bottoms!

*Trouble with this? Jump to: **Fractions of a Whole** (p. 64)*

If the denominators are not equal, they may be multiples of each other. We can make them equal using equivalent fractions.

**Key point**

Make the denominators equal using equivalent fractions, then add or subtract the numerators.

**Example 16.17**

$$\frac{2}{7} + \frac{4}{35} =$$

Change the first fraction to have the same denominator as the second fraction, using equivalent fractions:

$$\frac{2 \times 5}{7 \times 5} = \frac{10}{35}$$

Add the fractions:

$$\frac{10}{35} + \frac{4}{35} = \frac{14}{35}$$

Simplify the result, divide by 7:

$$\frac{14 \div 7}{35 \div 7} = \frac{2}{5}$$

$$\frac{2}{7} + \frac{4}{35} = \frac{2}{5}$$

Multiply the top and bottom of the second fraction by 6 to make the denominators equal:

$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

Subtract the fractions:

$$\frac{11}{18} - \frac{6}{18} = \frac{5}{18}$$

$$\frac{11}{18} - \frac{1}{3} = \frac{5}{18}$$

$$\frac{11}{18} - \frac{1}{3} =$$

**Questions B**

$$1. \frac{5}{7} + \frac{3}{14} =$$

$$3. \frac{10}{16} - \frac{5}{7} =$$

$$5. \frac{10}{24} + \frac{4}{12} =$$

$$7. \frac{2}{3} + \frac{9}{24} =$$

$$9. \frac{3}{8} - \frac{5}{24} =$$

$$2. \frac{2}{3} - \frac{4}{15} =$$

$$4. \frac{1}{3} + \frac{15}{32} =$$

$$6. \frac{4}{7} + \frac{42}{7} =$$

$$8. \frac{11}{11} - \frac{7}{33} =$$

$$10. \frac{3}{5} + \frac{4}{45} =$$

**Hints / Notes**

Change one of the fractions to match the denominator of the other. Remember to simplify your answer.

Trouble with this? Jump to: **Equivalent Fractions** (p. 71)

Some denominators are not equal and not multiples of each other. In these cases, we must change both denominators, using equivalent fractions. We find the *lowest common multiple* of both numbers, this is called the lowest common denominator.

**Key point**

Change both denominators using equivalent fractions, by finding the lowest common denominator.

### Example 16.18

$$\frac{5}{8} + \frac{4}{5} =$$

The lowest common multiple of 5 and 8 is 40.

Multiply the first fraction by 5 and the second fraction by 8, to make both denominators equal to 40:

$$\frac{5 \times 5}{8 \times 5} = \frac{25}{40}$$

$$\frac{4 \times 8}{5 \times 8} = \frac{32}{40}$$

Add the fractions:

$$\frac{25}{40} + \frac{32}{40} = \frac{57}{40}$$

The result is top heavy, change to a mixed number:

$$\frac{57}{40} = 1 \frac{17}{40}$$

$$\frac{5}{8} + \frac{4}{5} = 1 \frac{17}{40}$$

### Questions C

$$\begin{array}{l} 1. \frac{1}{2} + \frac{3}{4} = \\ 3. \frac{1}{3} - \frac{2}{5} = \\ 5. \frac{6}{7} + \frac{1}{11} = \\ 7. \frac{1}{2} - \frac{1}{11} = \end{array}$$

$$\begin{array}{l} 2. \frac{3}{4} - \frac{2}{9} = \\ 4. \frac{1}{6} + \frac{1}{9} = \\ 6. \frac{6}{8} - \frac{6}{10} = \\ 8. \frac{6}{13} + \frac{2}{3} = \end{array}$$

#### Hints / Notes

Change both fractions so that they have the same denominators. Add or subtract the tops and write over the bottoms.

*Trouble with this? Jump to: **Equivalent Fractions** (p. 71); **Lowest Common Multiple (LCM)** (p. 43); **Changing Improper Fractions to Mixed Numbers** (p. 78)*

## 16.11 Adding and Subtracting Mixed Numbers

**Grade 3** When adding or subtracting mixed numbers, we change each fraction to an improper fraction, then we add or subtract normally. An alternative method for adding mixed numbers is to add the whole numbers first, then the fractions. Both methods are shown in the examples.

#### Key point

Change each fraction to a top-heavy fraction, then add or subtract normally.

**Example 16.19**

$$1\frac{2}{3} + 3\frac{7}{9} =$$

Add the whole numbers.

$$1 + 3 = 4$$

Make the denominators of the remaining fractions equal, using equivalent fractions:

$$\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

Add the fractions:

$$\frac{6}{9} + \frac{7}{9} = \frac{13}{9}$$

Change the top-heavy fraction to a mixed number:

$$\frac{13}{9} = 1\frac{4}{9}$$

Add to the whole number:

$$4 + 1\frac{4}{9} = 5\frac{4}{9}$$

$$1\frac{2}{3} + 3\frac{7}{9} = 5\frac{4}{9}$$

Change any mixed numbers to improper fractions:

$$1\frac{3}{5} = \frac{8}{5}$$

Change the fraction using equivalent fractions:

$$\frac{8 \times 2}{5 \times 2} = \frac{16}{10}$$

$$\frac{16}{10} - \frac{9}{10} = \frac{7}{10}$$

$$1\frac{3}{5} - \frac{9}{10} = \frac{7}{10}$$

$$1\frac{3}{5} - \frac{9}{10} =$$

**Questions**

$$1. 3\frac{3}{4} + \frac{3}{7} =$$

$$3. 2\frac{1}{2} + 5\frac{3}{4} =$$

$$5. 4\frac{1}{2} - 2\frac{3}{4} =$$

$$7. 1\frac{4}{14} + 2\frac{3}{7} =$$

$$2. 2\frac{3}{8} - 1\frac{7}{10} =$$

$$4. 3\frac{1}{4} + 5\frac{2}{3} =$$

$$6. 1\frac{2}{3} - \frac{1}{7} =$$

$$8. 5\frac{3}{3} - \frac{8}{5} =$$

**Hints / Notes**

Improper fractions are top heavy. Mixed numbers are whole numbers with fractions. You can always change from mixed numbers to top heavy before adding or subtracting, then you can't go wrong!

*Trouble with this? Jump to: **Adding and Subtracting Fractions** (p. 81); **Changing Mixed Numbers to Improper Fractions** (p. 76)*

## 16.12 Multiplying Fractions

**Grade 3** Multiplying fractions is easier than adding them! We don't have to worry about the denominators (bottom) being the same. To multiply fractions, multiply the numerators (top) then multiply the denominators.

### Key point

Multiply the numerators then multiply the denominators.

### Example 16.20

$$\frac{7}{8} \times \frac{4}{5} =$$

Multiply the numerators:

$$7 \times 4 = 28$$

Multiply the denominators:

$$8 \times 5 = 40$$

$$= \frac{28}{40}$$

Simplify the answer:

$$\frac{28 \div 4}{40 \div 4} = \frac{7}{10}$$

$$\frac{7}{8} \times \frac{4}{5} = \frac{7}{10}$$

$$6 \times 7 = 42$$

Work out  $35 \times 12$ :

$$30 \times 12 = 360$$

$$5 \times 12 = 60$$

$$360 + 60 = 420$$

Write the answer as a fraction:

$$= \frac{42}{420}$$

Simplify the answer:

$$\frac{42 \div 42}{420 \div 42} = \frac{1}{10}$$

Alternative method:

The fractions can be simplified first, 6 and 12 can be divided by 6. 7 and 35 can be divided by 7. (Simplifying this way is only possible when multiplying fractions).

$$\frac{6}{35} \times \frac{7}{12} =$$

Multiply the fractions:

$$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

$$\frac{6}{35} \times \frac{7}{12} = \frac{1}{10}$$

$$\frac{6}{35} \times \frac{7}{12} =$$

## Questions

$$1. \frac{1}{3} \times \frac{3}{4} =$$

$$3. \frac{1}{2} \times \frac{4}{6} =$$

$$5. \frac{2}{5} \times \frac{10}{8} =$$

$$7. \frac{3}{10} \times \frac{16}{8} =$$

$$2. \frac{6}{7} \times \frac{3}{4} =$$

$$4. \frac{5}{7} \times \frac{3}{7} =$$

$$6. \frac{33}{14} \times \frac{7}{80} =$$

$$8. \frac{33}{14} \times \frac{7}{80} =$$

### Hints / Notes

You can simplify the answer by repeatedly dividing the top and bottom by 2, if the numbers remain even!

Trouble with this? Jump to: **Simplifying Fractions by Cancelling Down** (p. 74); **Changing Improper Fractions to Mixed Numbers** (p. 78)

## 16.13 Multiplying Mixed Numbers

**Grade 3** Mixed numbers must be changed to improper fractions before multiplying, then they can be multiplied normally.

### Key point

Change mixed numbers to top-heavy fractions before multiplying.

### Example 16.21

$$2\frac{2}{3} \times \frac{4}{11} =$$

Change  $2\frac{2}{3}$  to an improper fraction:

$$2\frac{2}{3} = \frac{8}{3}$$

Multiply the fractions:

$$\frac{8}{3} \times \frac{4}{11} = \frac{32}{33}$$

$$2\frac{2}{3} \times \frac{4}{11} = \frac{32}{33}$$

$$1\frac{5}{13} \times 3\frac{9}{10} =$$

Change  $1\frac{5}{13}$  and  $3\frac{9}{10}$  to improper fractions:

$$1\frac{5}{13} = \frac{18}{13} \text{ and } 3\frac{9}{10} = \frac{39}{10}$$

$$\frac{18}{13} \times \frac{39}{10} =$$

First Method:

Work out the numerator:  $18 \times 39$ :

$$\begin{array}{r} \times \quad 10 \quad 8 \\ 30 \quad 300 \quad 240 \\ 9 \quad 90 \quad 72 \end{array}$$

$$300 + 240 + 90 + 72 = 702$$

Work out the denominator:  $13 \times 10 = 130$

$$= \frac{702}{130}$$

Simplify the answer, divide by 2 then 13:

$$\frac{702}{130} = \frac{351}{65} = \frac{27}{5}$$

Alternative method:

Simplify first. 18 and 10 can be divided by 2.

13 and 39 can be divided by 13:

$$\frac{18^9}{13^1} = \frac{39^3}{10^5}$$

$$\frac{9}{1} \times \frac{3}{5} = \frac{27}{5}$$

Change to a mixed number:

$$\frac{27}{5} = 5\frac{2}{5}$$

$$1\frac{5}{13} \times 3\frac{9}{10} = 5\frac{2}{5}$$

## Questions

$$1. 2\frac{1}{4} \times 5\frac{1}{4} =$$

$$3. 3\frac{1}{4} \times 2\frac{3}{4} =$$

$$5. 5\frac{1}{2} \times 1\frac{1}{2} =$$

$$7. 3\frac{1}{6} \times 2\frac{2}{3} =$$

$$2. 2\frac{4}{5} \times 5\frac{5}{6} =$$

$$4. 3\frac{1}{2} \times 2\frac{1}{9} =$$

$$6. 1\frac{1}{2} \times 2\frac{2}{3} =$$

$$8. 6\frac{1}{2} \times 1\frac{10}{26} =$$

### Hints / Notes

Improper fractions are top heavy. Cancelling down the fractions first avoids large multiplications!

Trouble with this? Jump to: **Changing Mixed Numbers to Improper Fractions** (p. 76); **Multiplying Fractions** (p. 85)

## 16.14 Multiplying Fractions and Whole Numbers

**Grade 3** Sometimes we multiply a fraction by a single whole number, called an integer. Before multiplying, we change the integer into a fraction, then we multiply normally.

### Key point

Change integers into fractions, by writing over 1, before multiplying.

### Example 16.22

$$4 \times \frac{1}{7} =$$

4 is the same as 4 over 1. Change 4 into the fraction  $\frac{4}{1}$ :

$$\frac{4}{1} \times \frac{1}{7} = \frac{4}{7}$$

$$4 \times \frac{1}{7} = \frac{4}{7}$$

$$\frac{5}{8} \times 3 =$$

3 is the same as 3 over 1. Change 3 into the fraction  $\frac{3}{1}$ :

$$\frac{5}{8} \times \frac{3}{1} = \frac{15}{8}$$

Change the answer to a mixed number:

$$\frac{15}{8} = 1\frac{7}{8}$$

$$\frac{5}{8} \times 3 = 1\frac{7}{8}$$

## Questions

$$1. \frac{3}{5} \times 11 =$$

$$3. \frac{3}{5} \times \frac{2}{13} =$$

$$5. \frac{4}{7} \times 8 =$$

$$7. 6 \times \frac{4}{9} =$$

$$2. \frac{5}{8} \times 4 =$$

$$4. 10 \times \frac{6}{11} =$$

$$6. \frac{5}{6} \times 7 =$$

$$8. 5 \times \frac{3}{8} =$$

### Hints / Notes

Change whole numbers into fractions before multiplying and dividing! Fractions mean divide. For example,  $\frac{3}{2}$  means 3 divided by 2.

Trouble with this? Jump to: **Multiplying Fractions** (p. 85); **Changing Improper Fractions to Mixed Numbers** (p. 78)

## 16.15 Dividing Fractions

**Grade 3** When dividing fractions, we remember the phrase: Keep Change Flip

Keep the first fraction.

Change the  $\div$  sign to  $\times$ .

Flip the second fraction.

### Key point

Keep the first fraction, change the divide sign to multiply, flip the second fraction.

### Example 16.23

$$\frac{3}{8} \div \frac{2}{5} =$$

Keep the first fraction  $\frac{3}{8}$ .

Change the  $\div$  sign to multiply  $\times$ .

Flip the second fraction,  $\frac{2}{5}$ , the top becomes the bottom and the bottom becomes the top,

$$\frac{5}{2} :$$

$$\frac{3}{8} \times \frac{5}{2} =$$

Multiply normally:

$$\frac{3}{8} \times \frac{5}{2} = \frac{15}{16}$$

$$\frac{4}{9} \div \frac{1}{7} =$$

$$\frac{3}{8} \div \frac{2}{5} = \frac{15}{16}$$

Keep the first fraction  $\frac{4}{9}$ .

Change the  $\div$  sign to multiply  $\times$ .

Flip the second fraction  $\frac{1}{7}$ , the top becomes the bottom and the bottom becomes the top,

$$\frac{7}{1} :$$

$$\frac{4}{9} \times \frac{7}{1} =$$

Multiply normally:

$$\frac{4}{9} \times \frac{7}{1} = \frac{28}{9}$$

If the answer is top heavy, change it to a mixed number:

$$\frac{28}{9} = 3\frac{1}{9}$$

$$\frac{4}{9} \div \frac{1}{7} = 3\frac{1}{9}$$

### Questions

$$1. \frac{5}{7} \div \frac{10}{23} =$$

$$3. \frac{10}{7} \div \frac{10}{7} =$$

$$5. \frac{8}{5} \div \frac{3}{5} =$$

$$7. \frac{8}{3} \div \frac{3}{4} =$$

$$2. \frac{3}{4} \div \frac{7}{9} =$$

$$4. \frac{4}{9} \div \frac{9}{18} =$$

$$6. \frac{7}{9} \div \frac{2}{12} =$$

$$8. \frac{7}{9} \div \frac{2}{45} =$$

### Hints / Notes

Remember: Keep Change Flip!

Trouble with this? Jump to: **Multiplying Fractions** (p. 85)

## 16.16 Dividing Mixed Numbers

**Grade 3** Before dividing mixed numbers, we change them to improper fractions. We divide normally, remembering to keep, change, flip.

### Key point

Change mixed numbers to top-heavy fractions before dividing.

### Example 16.24

$$3\frac{4}{5} \div \frac{2}{3} =$$

Change  $3\frac{4}{5}$  into an improper fraction:

$$3\frac{4}{5} = \frac{19}{5}$$

$$\frac{19}{5} \div \frac{2}{3} =$$

Divide the fractions (keep, change, flip):

$$\frac{19}{5} \times \frac{3}{2} = \frac{57}{10}$$

$$2\frac{1}{4} \div 4\frac{2}{5} =$$

If the answer is top-heavy, change to a mixed number:

$$\frac{57}{6} = 9\frac{3}{6}$$

Simplify the fraction:

$$9\frac{3}{6} = 9\frac{1}{2}$$

$$3\frac{4}{5} \div \frac{2}{3} = 9\frac{1}{2}$$

Change  $2\frac{1}{4}$  and  $4\frac{2}{5}$  into improper fractions:

$$2\frac{1}{4} = \frac{9}{4}$$

$$4\frac{2}{5} = \frac{22}{5}$$

$$\frac{9}{4} \div \frac{22}{5} =$$

Divide the fractions, (keep, change, flip):

$$\frac{9}{4} \times \frac{5}{22} = \frac{45}{88}$$

## Questions

$$1. 1\frac{1}{3} \div \frac{4}{5} =$$

$$3. 3\frac{3}{4} \div 1\frac{1}{2} =$$

$$5. 2\frac{1}{2} \div 5\frac{6}{7} =$$

$$7. 1\frac{3}{8} \div 4\frac{4}{5} =$$

$$2. 7\frac{1}{2} \div 5\frac{1}{6} =$$

$$4. \frac{4}{9} \div 2\frac{4}{6} =$$

$$6. 4\frac{1}{2} \div 3\frac{1}{3} =$$

$$8. \frac{1}{18} \div 1\frac{2}{9} =$$

### Hints / Notes

First change to improper fractions, then keep change flip!

Trouble with this? Jump to: **Changing Mixed Numbers to Improper Fractions** (p. 76)

## 16.17 Dividing Fractions and Whole Numbers

**Grade 3** Sometimes we divide with fractions and whole numbers, called integers. Before dividing, we change the integer into a fraction, then we divide normally.

### Key point

Change integers into fractions before dividing.

### Example 16.25

$$\frac{2}{9} \div 5 =$$

5 is the same as  $\frac{5}{1}$ . Change the 5 into a fraction.

$$\frac{2}{9} \div \frac{5}{1} =$$

Divide normally:

$$\frac{2}{9} \times \frac{1}{5} = \frac{2}{45}$$

$$7 \div \frac{2}{5} =$$

$$\frac{2}{9} \div 5 = \frac{2}{45}$$

7 is the same as  $\frac{7}{1}$ . Change the 7 to a fraction.

$$\frac{7}{1} \div \frac{2}{5} =$$

Divide normally. If the answer is top heavy, change it to a mixed number.

$$\frac{7}{1} \times \frac{5}{2} = \frac{35}{2}$$

$$\frac{35}{2} = 17\frac{1}{2}$$

$$7 \div \frac{2}{5} = 17\frac{1}{2}$$

## Questions

$$1. \frac{5}{7} \div 6 =$$

$$3. 7 \div \frac{2}{3} =$$

$$5. \frac{4}{5} \div \frac{1}{5} =$$

$$7. 8 \div \frac{1}{4} =$$

$$2. \frac{7}{11} \div 3 =$$

$$4. 9 \div \frac{3}{4} =$$

$$6. \frac{5}{6} \div \frac{2}{5} =$$

$$8. 4 \div \frac{2}{9} =$$

## Hints / Notes

Change whole numbers into fractions before multiplying and dividing! Write 6 as  $\frac{6}{1}$ , it means the same thing! Remember to keep, change, flip!

Trouble with this? Jump to: **Changing Mixed Numbers to Improper Fractions** (p. 76); **Multiplying Fractions** (p. 85)

## 16.18 Percentages

**Grade 3** Fractions, decimal numbers and percentages are different ways of writing parts of a number. One quarter can be written as a fraction  $\frac{1}{4}$ , a decimal 0.25, or a percentage 25%.

100% of something means all of it, if I watched 100% of a film, then I watched all of it. 50% of something means half of it, if I cycled 50% of the way to school then I cycled half of the distance.

### Key point

100% of something is the whole thing, 50% is half, 25% is a quarter.

### Example 16.26

Sarah has a box of chocolates,

$$100 - 70 = 30$$

30% are plain.

70% of them are milk chocolates and the rest are plain. What percentage are plain?

Lorenzo watched 85% of his gardening programme.  $100 - 85 = 15$   
Lorenzo missed 15%.

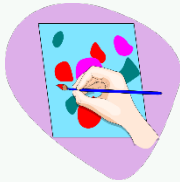
What percentage did he miss?

45% of Tina's class are girls, what percentage are boys?  $100 - 45 = 55$   
55% are boys.

## Questions

1. Callum's Mum makes celebration cards. 60% of the cards are birthday cards, what percentage are not birthday cards?

2. Maya likes to paint and sketch. 25% of Maya's pictures are paintings. What percentage



are sketches?

3. Toby runs 72% of his racing track. What percentage does he have left to run?

4. Rick eats 33% of his pizza. What percentage does Rick have left?

5. Pippa keeps animals on her farm. 65% of the animals are put into the barn at night, what percentage are left outside?

6. 6% of the world's population speak English as their first language. What percentage do not speak English as their first language?

7. The amount of water in a human body is 57%. What percentage is not water?

8. Our atmosphere is made up of 78% nitrogen and 21% oxygen. The rest is made up of other gases. What percentage are the other gases?

## Hints / Notes

Take the percentage away from 100 to find out what's left!

## 16.19 Decimal Numbers and Percentages

**Grade 3** To change from a decimal number to a percentage, we multiply by 100. We can move the digits 2 places to the left or we can use a calculator.

### Key point

To change from a decimal to a percentage, multiply by 100.

### Example 16.27

$0.14 \times 100 = 14 = 14\%$       Move the digits 2 places to the left.

$$2.57 \times 100 = 257 = 257\% \quad \text{Move the digits 2 places to the left.}$$

### Questions A

Change these decimal numbers to percentages.

- |         |          |
|---------|----------|
| 1. 0.31 | 2. 0.45  |
| 3. 0.5  | 4. 0.65  |
| 5. 0.64 | 6. 0.21  |
| 7. 0.07 | 8. 0.93  |
| 9. 2.75 | 10. 1.25 |

#### Hints / Notes

Some decimal numbers are greater than 1, for example, 1.5 means one whole one plus one half which is 150%.

*Trouble with this? Jump to: **Multiplying Decimal Numbers by Powers of Ten** (p. 153)*

To change from a percentage to a decimal number, we divide by 100. We think of the number as having a decimal point and a zero at the end then we move the digits 2 places to the right over the decimal point. We could also use a calculator.

#### Key point

To change from a percentage to a decimal, divide by 100.

#### Example 16.28

$25\% : 25 \div 100 = 0.25$     Imagine the number as being 25.0 then move the digits 2 places to the right, over the decimal point.

$120\% : 120 \div 100 = 1.2$     Imagine the number as being 120.0 then move the digits 2 places to the right, over the decimal point.

### Questions B

Change these percentages to decimal numbers.

- |         |          |
|---------|----------|
| 1. 33%  | 2. 17%   |
| 3. 20%  | 4. 23%   |
| 5. 75%  | 6. 78%   |
| 7. 50%  | 8. 55%   |
| 9. 66%  | 10. 8%   |
| 11. 6%  | 12. 97%  |
| 13. 80% | 14. 113% |

#### Hints / Notes

Some percentages are greater than 100, this means more than one whole. For example,

*Trouble with this? Jump to: **Dividing Whole Numbers by Powers of Ten** (p. 151)*

230% means two whole ones plus 30% which is 2.3 as a decimal.

## 16.20 Percentages and Fractions

Grade 3

### Percentages to Fractions

Percent means out of 100. To change from a percentage to a fraction, we write the number over 100, then we simplify the fraction.

#### Key point

To change from a percentage to a fraction, write the percentage over 100, then simplify the fraction.

#### Example 16.29

$12\% = \frac{12}{100} = \frac{3}{25}$  Write 12 over 100, then simplify by dividing the top and bottom by 4.  
 $35\% = \frac{35}{100} = \frac{7}{20}$  Write 35 over 100, then simplify by dividing the top and bottom by 5.

### Questions

Change these percentages to fractions.

- |        |        |
|--------|--------|
| 1. 18% | 2. 50% |
| 3. 45% | 4. 80% |
| 5. 84% | 6. 75% |
| 7. 30% | 8. 20% |

*Trouble with this? Jump to: [Simplifying Fractions by Cancelling Down](#) (p. 74)*

### Fractions to Percentages

#### Method 1

We can multiply the top and the bottom of a fraction by a number, so that the denominator (bottom) becomes one hundred. Then we can write the numerator (top) as the percentage.

#### Key point

Multiply the fraction so that the denominator becomes one hundred, the answer is the numerator.

#### Example 16.30

$\frac{12}{50} = \frac{12 \times 2}{50 \times 2} = \frac{24}{100} = 24\%$  Make the denominator equal to 100 by multiplying the fraction by 2. 24 out of 100.

$$\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100} = 36\% \quad \text{Make the denominator equal to 100 by multiplying the fraction by 4. 36 out of 100.}$$

$$\frac{11}{20} = \frac{11 \times 5}{20 \times 5} = \frac{55}{100} = 55\% \quad \text{Make the denominator equal to 100 by multiplying the fraction by 5. 55 out of 100.}$$

### Questions A

Change these fractions to percentages.

1.  $\frac{11}{50}$
3.  $\frac{1}{20}$
5.  $\frac{29}{100}$
7.  $\frac{25}{100}$
9.  $\frac{49}{50}$

2.  $\frac{7}{25}$
4.  $\frac{3}{10}$
6.  $\frac{5}{6}$
8.  $\frac{29}{100}$
10.  $\frac{19}{20}$

#### Hints / Notes

Write the fraction as something out of 100.  
Anything out of one hundred is a percentage!

Trouble with this? Jump to: **Equivalent Fractions** (p. 71)

### Method 2

To change from a fraction to a percentage, we can multiply the numerator (top) by 100, then divide by the denominator (bottom). We can use the bus stop method or a calculator to divide.

#### Key point

To change a fraction to a percentage, multiply the numerator by 100 then divide by the denominator.

#### Example 16.31

$$\frac{1}{5} : 1 \times 100 = 100, 100 \div 5 = 20 = 20\% \quad \text{Multiply the numerator by 100, then divide by the denominator.}$$

$$\frac{3}{4} : 3 \times 100 = 300, 300 \div 4 = 75 = 75\% \quad \text{Multiply the numerator by 100, then divide by the denominator.}$$

### Questions B

Change these fractions to percentages.

1.  $\frac{3}{100}$
3.  $\frac{1}{20}$
5.  $\frac{7}{100}$
7.  $\frac{4}{10}$
9.  $\frac{4}{5}$

2.  $\frac{1}{10}$
4.  $\frac{3}{10}$
6.  $\frac{4}{5}$
8.  $\frac{10}{100}$
10.  $\frac{3}{20}$

**Hints / Notes**

Remember how to do the bus stop method?

If not jump to the topic!

Trouble with this? Jump to: **Dividing Whole Numbers by Single Digits** (p. 178); **Dividing Whole Numbers by Powers of Ten** (p. 151)


We can use a calculator to change fractions into percentages. Enter the fraction into the calculator, press the S↔D button to display it as a decimal then multiply by 100. If the answer has lots of digits, we can round to 1 or 2 decimal places.

**Key point**

Enter the fraction into the calculator, press S↔D then multiply by 100.

**Example 16.32**

$$\frac{12}{62}$$


Enter  $\frac{12}{62}$  into the calculator using the fraction button .

Press the S↔D button: = 0.193548...

Multiply by 100:  $0.193548... \times 100 = 19.3548...$

Round to 1 decimal place: = 19.4% (1 d.p.)

$$\frac{9}{28}$$

Enter  $\frac{9}{28}$  into the calculator using the fraction button .

Press the S↔D button: = 0.32142...

Multiply by 100:  $0.32142... \times 100 = 32.142...$

Round to 1 decimal place: = 31.2% (1 d.p.)

**Questions C**

Change these fractions to percentages.

1.  $\frac{3}{11}$
3.  $\frac{24}{24}$
5.  $\frac{16}{25}$
7.  $\frac{48}{45}$

2.  $\frac{7}{26}$
4.  $\frac{88}{88}$
6.  $\frac{99}{99}$
8.  $\frac{28}{31}$
10.  $\frac{28}{31}$

**Hints / Notes**

Remember to press [=] after entering the fraction with . Round the answer to 1 or 2 decimal places.

To solve some problems, we need to change fractions into percentages.

**Example 16.33**

Sam got 43 out of 50 for his History exam. What percentage is this?

Write 43 out of 50 as a fraction:  $= \frac{43}{50}$

Multiply the fraction by 2 to make the denominator 100:  $\frac{43 \times 2}{50 \times 2} = \frac{86}{100}$

Sam got 86%.

Ella picked 8 girls out of a class of 20 for her reading group. What percentage did she choose?

Write 8 out of 20 as a fraction:  $= \frac{8}{20}$

Multiply by 5 to make the denominator 100:  $\frac{8 \times 5}{20 \times 5} = \frac{40}{100}$

Ella chose 40% of the class.

Edward's chocolate box had 75 chocolates. Edward ate 23 of the chocolates. What percentage did Edward eat?

Write 23 out of 75 as a fraction:  $= \frac{23}{75}$

Enter the fraction into the calculator using the fraction button.

Press the S $\leftrightarrow$ D button to display as a decimal:  $= 0.30666666...$

Multiply by 100:  $0.30666666... \times 100 = 30.666666...%$

Round the answer to 1 decimal place: Edward ate 30.7% of the chocolates. (1 d.p.)

**Questions D**

Change these fractions to percentages, use a calculator if necessary.

- 4 out of 10 cats are tabby. What percentage are tabby?
- Gita picked out 21 cakes for her coffee morning out of the 25 on display. What percentage did she pick?
- Tamil has 12 odd socks out of 20. What percentage of Tamil's socks are odd?
- Billy jumps 14 out of 30 hurdles in a race. What percentage did he jump?
- Stephanie plays 12 out of 15 melodies from her music book. What percentage did she



play?

- Max's hockey team has 18 players but 4 of them are sick. What percentage are sick?
- 32 out of 50 workers at a factory are women. What percentage are women?
- 19 people out of 45 voted for a new supermarket in their town. What percentage voted?

**Hints / Notes**

Trouble with this? Jump to: **Fractions to Percentages** (p. 94)

Remember to press [=] after entering the fraction with  $\frac{\square}{\square}$ , otherwise the fraction is not yet in the calculator!

## 16.21 Decimal Numbers and Fractions

Grade 3

### Decimal Numbers to Fractions

To change a decimal number to a fraction, we write it as a whole number, divided by a power of 10. Then we simplify the fraction. For example, 0.6 is 6 divided by 10, 0.07 is 7 divided by 100. To find out what to divide by, multiply the number by ten until it is a whole number, then divide by the same amount.

#### Key point

To change a decimal to a fraction, write it as a whole number divided by 10, 100 or 1000.

#### Example 16.34

0.2

0.2 is the same as 2 divided by 10. Write as a fraction:  $= \frac{2}{10}$

Simplify the fraction by dividing the top and bottom by 2:  $= \frac{1}{5}$

0.34

0.34 is the same as 34 divided by 100. Write as a fraction:  $= \frac{34}{100}$

Simplify the fraction by dividing the top and bottom by 2:  $= \frac{17}{50}$

0.782

0.782 is the same as 782 divided by 1000. Write as a fraction:  $= \frac{782}{1000}$

Simplify the fraction by dividing the top and bottom by 2:  $= \frac{391}{500}$

2.6

2.6 is the same as 26 divided by 10. Write as a fraction:  $= \frac{26}{10}$

This fraction is top heavy, change it to a mixed number, by dividing the top by the bottom:  $26 \div 10 = 2$  remainder 6, so  $= 2\frac{6}{10}$

Simplify the fraction by dividing by 2:  $2\frac{6}{10} = 2\frac{3}{5}$

## Questions

Change these decimal numbers to fractions.

1. 0.8

3. 0.73

5. 0.92

7. 0.75

9. 1.4

2. 0.15

4. 0.3

6. 0.24

8. 0.855

10. 2.85

### Hints / Notes

The first three powers of 10 are 10, 100 and 1000.

*Trouble with this? Jump to: **Dividing Whole Numbers by Powers of Ten** (p. 151); **Simplifying Fractions by Cancelling Down** (p. 74); **Changing Improper Fractions to Mixed Numbers** (p. 78)*

## Fractions to Decimal Numbers

To change a fraction to a decimal, we divide the numerator (top) by the denominator (bottom). If the denominator is a power of ten, then we can divide more easily.

### Key point

To change a fraction to a decimal, divide the numerator by the denominator.

### Example 16.35

$$\frac{6}{10}$$

Divide the numerator by the denominator:

$$6 \div 10 = 0.6$$

$$\frac{11}{20}$$

Change the denominator to 100 then we can divide more easily. Multiply the fraction by 5 using equivalent fractions:

$$\frac{11 \times 5}{20 \times 5} = \frac{55}{100}$$

$$55 \div 100 = 0.55$$

$$\frac{1}{5}$$

Change the denominator to 10 then we can divide more easily. Multiply the fraction by 2 to make the denominator equal to 10:

$$\frac{1 \times 2}{5 \times 2} =$$

$$\frac{2}{10}$$

$$2 \div 10 = 0.2$$

## Questions A

Change these fractions to decimal numbers.

1.  $\frac{1}{10}$

3.  $\frac{7}{10}$

5.  $\frac{3}{10}$

7.  $\frac{11}{25}$

2.  $\frac{7}{20}$

4.  $\frac{25}{100}$

6.  $\frac{13}{20}$

8.  $\frac{20}{5}$

**Hints / Notes**

The first three powers of 10 are 10, 100 and 1000.

*Trouble with this? Jump to: **Dividing Whole Numbers by Powers of Ten** (p. 151)*

If the fraction is a mixed number, then the decimal will be the whole number plus the new decimal part. Change the fraction to a decimal first, then add the whole number.

**Key point**

Change the fraction to a decimal then add on the whole number.

**Example 16.36**

$$3\frac{7}{10}$$

Change the fraction part to a decimal:  $\frac{7}{10} = 0.7$

Add the whole number:  $3 + 0.7 = 3.7$

$$6\frac{12}{25}$$

Change the fraction part to a decimal:  $\frac{12}{25}$

Multiply the fraction by 4 to get 100 on the bottom:  $\frac{12 \times 4}{25 \times 4} = \frac{48}{100}$

Change the fraction to a decimal:  $\frac{48}{100} = 0.48$

Add the whole number:  $6 + 0.48 = 6.48$

**Questions B**

Change these fractions to decimal numbers.

1.  $1\frac{3}{10}$
3.  $4\frac{10}{100}$
5.  $2\frac{1}{100}$
7.  $3\frac{2}{5}$

2.  $8\frac{17}{25}$
4.  $7\frac{1}{5}$
6.  $4\frac{2}{20}$
8.  $6\frac{13}{20}$

**Hints / Notes**

Change the fraction to a decimal then add on the whole number.

*Trouble with this? Jump to: **Fractions to Decimal Numbers** (p. 99)*

If the denominator (bottom) is not a multiple of ten, then we can use the bus stop method to divide the numerator (top) by the denominator. We write the numerator as a decimal, adding more zeros as we need them.

**Key point**

Divide the numerator by the denominator using the bus stop method.

**Example 16.37** $\frac{7}{8}$ 

Think of 7 as being 7.0 then write the fraction as a division, using the bus stop method.

$$8 \overline{) 7.0}$$

8 goes into 7 zero times with remainder 7. Place 0 on the top line followed by a decimal point. Place 7 in front of the next digit, zero becomes 70.

$$\begin{array}{r} 0.875 \\ 8 \overline{) 7.000} \\ \underline{7 \phantom{0} \phantom{0} \phantom{0}} \\ 6 \phantom{0} \phantom{0} \\ \underline{6 \phantom{0} \phantom{0}} \\ 4 \phantom{0} \\ \underline{4 \phantom{0}} \\ 0 \end{array}$$

8 goes into 70 eight times with remainder six. Place 8 on the top line. Add a zero to the number below. Place 6 in front of the next digit, zero. The zero becomes 60.

8 goes into 60 seven times with remainder 4. Place 7 on the top line. Add a zero to the number below. Place 4 in front of the next digit, zero. The zero becomes 40.

8 goes into 40 five times. Place 5 on the top line.

$$= 0.875$$

Think of 4 as being 4.0, then write the fraction as a division using the bus stop method.

$$7 \overline{) 4.0}$$

7 goes into 4 zero times with remainder 4. Place 0 on the top line. Place 4 in front of the next digit. The zero becomes 40.

$$\begin{array}{r} 0.5714 \\ 7 \overline{) 4.0000} \\ \underline{4 \phantom{0} \phantom{0} \phantom{0} \phantom{0}} \\ 0 \phantom{0} \phantom{0} \phantom{0} \\ \underline{0 \phantom{0} \phantom{0} \phantom{0}} \\ 0 \phantom{0} \phantom{0} \\ \underline{0 \phantom{0}} \\ 0 \end{array}$$

7 goes into 40 five times with 5 remainder. Place 5 on the top line. Add more zeros to

 $\frac{4}{7}$

the number below. Place 5 in front of the next digit.

7 goes into 50 seven times with remainder 1. Place 7 on the top line. Place 1 in front of the next digit. Zero becomes 10.

7 goes into 10 once with remainder 3. Place 1 on the top line. Place 3 in front of the next digit. Zero becomes 30.

7 goes into 30 four times with remainder 2. Place 4 on the top line.

We realise that this calculation keeps going! We choose to stop after four decimal places. We round the answer to 3 decimal places.

= 0.571 (3 d.p.)

### Questions C

Change these fractions to decimal numbers.

1.  $\frac{1}{2}$
3.  $\frac{1}{4}$
5.  $\frac{1}{8}$
7.  $\frac{1}{16}$

2.  $\frac{1}{3}$
4.  $\frac{1}{6}$
6.  $\frac{1}{12}$
8.  $\frac{1}{11}$

#### Hints / Notes

Round the answer to 3 decimal places, unless the question tells you a different amount.

*Trouble with this? Jump to: **Dividing Whole Numbers by Single Digits** (p. 178)*

We can use a calculator to change a fraction to a decimal. We enter the fraction into the calculator, then we press the  $S \leftrightarrow D$  button to display the fraction as a decimal. If the decimal has many digits, we round the answer to one or two decimal places.

#### Key point

Enter the fraction into the calculator, then press  $S \leftrightarrow D$ .

#### Example 16.38

$$\frac{55}{80}$$

Enter the fraction into the calculator using the fraction button.

$$2\frac{12}{35}$$

Press the S↔D button to display as a decimal.  
= 0.6875

Round the answer to 3 decimal places: =  
0.688

Enter the fraction into the calculator using the mixed fraction button.

Press the S↔D button to display as a decimal.  
= 2.342857

Round the answer to 3 decimal places: =  
2.343

### Questions D

Change these fractions to decimal numbers.

1.  $\frac{12}{47}$
3.  $\frac{38}{15}$
5.  $\frac{26}{85}$
7.  $\frac{8}{87}$
9.  $\frac{16}{87}$

2.  $\frac{12}{28}$
4.  $\frac{47}{80}$
6.  $\frac{1}{15}$
8.  $1\frac{13}{69}$
10.  $2\frac{61}{90}$

#### Hints / Notes

The mixed fraction button is [SHIFT] then the fraction button. Remember to press [=] after entering the fraction.

## 16.22 Percentages of Shapes

**Grade 3** To find the percentage of an object that is shaded, find the fraction that is shaded, then change the fraction to a percentage. The fraction that is shaded is the number of shaded parts over the total number of parts.

Two methods of changing the fraction to a percentage are shown here. The first method changes the fraction to something over 100, using equivalent fractions. The second method finds a fraction of 100 by dividing. Choose the method that is most suitable for the question.

#### Key point

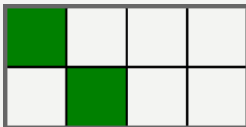
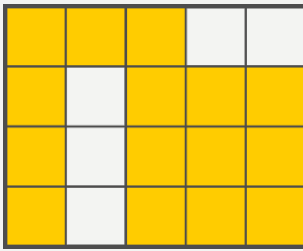
Find the fraction that is shaded, then change it to a percentage.

#### Example 16.39



The shaded part is  $\frac{1}{10}$ .

Use equivalent fractions:



$$\frac{1 \times 10}{10 \times 10} = \frac{10}{100} = 10\%$$

The shaded part is  $\frac{15}{20}$ .

Use equivalent fractions:

$$\frac{15 \times 5}{20 \times 5} = \frac{75}{100} = 75\%$$

The shaded part is  $\frac{2}{6}$ . Simplify the fraction:

$$\frac{2}{6} = \frac{1}{3}$$

Find  $\frac{1}{3}$  of 100:

$$100 \div 3 = 33.33333\dots$$

Round the answer to 1 decimal place:

$$33.3\%$$

The shaded part is  $\frac{2}{8}$ . Simplify the fraction:

$$\frac{2}{8} = \frac{1}{4}$$

Find  $\frac{1}{4}$  of 100:

$$100 \div 4 = 25\%$$

The shaded part is  $\frac{2}{9}$ .

Find  $\frac{2}{9}$  of 100:

$$2 \times 100 = 200$$

$$200 \div 9 = 22.2222\dots$$

Round to 1 decimal place:

$$= 22.2\%$$

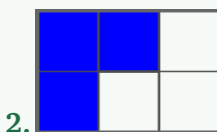
### Hints / Notes

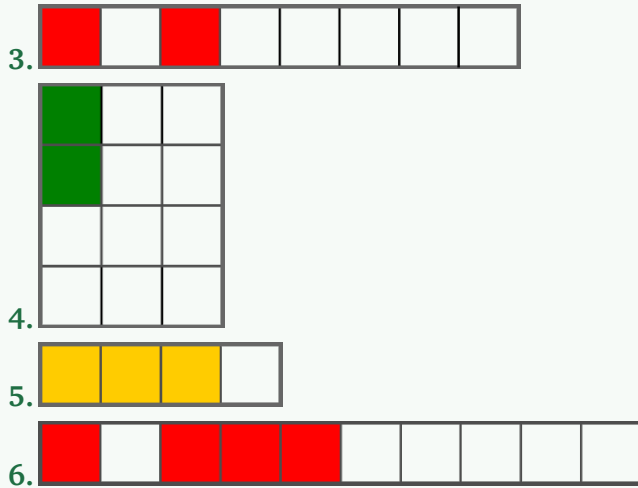
To find  $\frac{2}{9}$  of 100, you can also use the fraction button on your calculator to calculate  $\frac{2}{9} \times 100 =$

*Trouble with this? Jump to: **Fractions of Shapes** (p. 66); **Fractions to Percentages** (p. 94); **Simplifying Fractions by Cancelling Down** (p. 74)*

### Questions

Find the percentage that is shaded for these shapes.





### 16.23 Percentages of a Quantity Non-Calculator Method

**Grade 3** To find 10% of a quantity we divide the quantity by 10. We divide by 10 by moving the digits 1 place to the right over the decimal point. If the quantity is a whole number, then we can think of the number as having a point zero on the end. If the whole number ends in zero, then we can simply take off the zero.

#### Key point

To find 10% of a quantity, divide it by 10.

#### Example 16.40

10% of £300

As this is a whole number ending in zero, take off the last zero:

$$300 \div 10 = 30$$

$$= \text{£}30$$

10% of 54g

Think of the number as 54.0:

54.0

Move the digits 1 place to the right, over the decimal point:

$$54.0 \div 10 = 5.4$$

$$= 5.4 \text{ g}$$

10% of £4.76

Move the digits 1 place to the right, over the decimal point:

$$4.76 \div 10 = 0.476$$

Write the answer as pounds and pence, by rounding to 2 decimal places:

$$= \text{£}0.48$$

### Questions A

1. 10% of £400
3. 10% of £90
5. 10% of £32.60
7. 10% of £60.50
9. 10% of £12.08

2. 10% of £58.60
4. 10% of 27kg
6. 10% of 48.7ml
8. 10% of 34g
10. 10% of 89m

#### Hints / Notes

To change to pounds and pence, we round the number to 2 decimal places.

*Trouble with this? Jump to: **Dividing Decimal Numbers by Powers of Ten** (p. 154); **Rounding to Decimal Places** (p. 5)*

To find 20% of a quantity, we find 10% of the quantity, then we double it. To find 30% of a quantity, we find 10%, then we multiply by three. To find 90%, we can subtract 10% from the original quantity.

#### Key point

To find 20%, we find 10% then double it.

#### Example 16.41

20% of £75

Find 10%:

$$75 \div 10 = 7.5$$

Find 20%:

$$7.5 \times 2 = 15$$

$$= \text{£}15$$

40% of £34.50

Find 10%:

$$34.50 \div 10 = 3.45$$

Find 40%:

$$3.45 \times 4 = 13.8$$

Write as pounds and pence:

$$= \text{£}13.80$$

90% of 540 people.

Find 10%:

$$540 \div 10 = 54$$

Find 90%:

$$540 - 54 = 486$$

$$= 486 \text{ people.}$$

### Questions B

1. 20% of £300
3. 40% of £68
5. 40% of £80
7. 90% of 62kg
9. 80% of 75ml

2. 20% of 37ml
4. 30% of 60 bottles
6. 30% of 16km
8. 90% of 39cm
10. 70% of £32.40

#### Hints / Notes

To change to pounds and pence, we round the number to 2 decimal places.

*Trouble with this? Jump to: **Percentages of a Quantity Non-Calculator Method** (p. 105); **Rounding to Decimal Places** (p. 5); **Multiplying with Decimal Numbers** (p. 47)*

To find 15% of a quantity, we find 10%, then we halve it to find 5%. We add the values for 10% and 5%. We can find 35% for example, by finding 30%, then adding 5%.

#### Key point

To find 15%, we find 10%, halve it to get 5%, then add it to 10%.

#### Example 16.42

15% of £26

Find 10%:

$$26 \div 10 = 2.6$$

Find 5%:

$$2.6 \div 2 = 1.3$$

Find 15%:

$$2.6 + 1.3 = 3.9$$

$$= \text{£}3.90$$

35% of 437cm

Find 10%:

$$437 \div 10 = 43.7$$

Find 30%:

$$43.7 \times 3 = 131.1$$

Find 5%:

$$43.7 \div 2 = 21.85$$

Find 35%:

$$131.1 + 21.85 = 152.95$$

$$= 152.95\text{cm}$$

## Questions C

1. 15% of £3.80
2. 15% of £21.60
3. 25% of 58kg
4. 15% of £2100
5. 35% of £80
6. 35% of 18m
7. 45% of 340g
8. 55% of 65L
9. 95% of 87mm
10. 35% of £56.70

### Hints / Notes

A quicker way of finding 25% is to divide by

4. 25% means a quarter of it! To find 95% you can find 5% and take it away from 100!

*Trouble with this? Jump to: **Dividing Decimal Numbers by Powers of Ten** (p. 154); **Rounding to Decimal Places** (p. 5); **Dividing with Decimal Numbers** (p. 48)*

To find 10% of a number, we divide by 10. To find 1% of the number, we divide by 10 again. When we know 10% and 1% of a number, we can find other percentages by using these values. For example, to find 3%, we multiply 1% by 3.

### Key point

To find 1% of the number, divide by 10 twice.

### Example 16.43

11% of 924g

Find 10%:

$$924 \div 10 = 92.4$$

Find 1%:

$$92.4 \div 10 = 9.24$$

Find 11%:

$$\begin{aligned} 92.4 + 9.24 &= 101.64 \\ &= 101.64 \text{ g} \end{aligned}$$

13% of £64

Find 10%:

$$64 \div 10 = 6.4$$

Find 1%:

$$6.4 \div 10 = 0.64$$

Find 3%:

$$0.64 \times 3 = 1.92$$

Find 13%:

$$\begin{aligned} 6.4 + 1.92 &= 8.32 \\ &= \text{£}8.32 \end{aligned}$$

27% of 800m

Find 20%:

$$10\% = 800 \div 10 = 80$$

$$80 \times 2 = 160$$

Find 5%:

$$80 \div 2 = 40$$

Find 2%:

$$1\% = 80 \div 10 = 8$$

$$8 \times 2 = 16$$

Find 27%:

$$160 + 40 + 16 = 216$$

$$= 216 \text{ m}$$

### Questions D

1. 11% of 470m
3. 12% of £380
5. 21% of 290km
7. 31% of £7.40
9. 13% of £750
11. 24% of 55kg

2. 51% of 860g
4. 26% of 1700L
6. 42% of 360mm
8. 16% of £35.60
10. 32% of £650
12. 21% of 208m

#### Hints / Notes

90% is the original amount, minus 10%.

50% is half of the amount, 25% is a quarter of it.

*Trouble with this? Jump to: **Dividing Decimal Numbers by Powers of Ten** (p. 154)*

## 16.24 Percentage Increase and Decrease Non-Calculator Method

**Grade 3** To find the percentage increase of a quantity, we find the value of the increase, then we add it to the original amount. To find a percentage decrease, we find the value of the decrease then we subtract it from the original amount. An increase is also called a gain, a decrease is also called a loss.

#### Key point

Find the percentage then add to, or subtract from, the original amount.

#### Example 16.44

A gain of 5% on £40

Find 5% of 40:

$$10\% = 40 \div 10 = 4$$

$$5\% = 4 \div 2 = 2$$

An increase of 12% on 640L

Add to the original amount:

$$40 + 2 = 42$$

$$= \text{£}42.00$$

Find 12% of 640:

$$10\% = 640 \div 10 = 64$$

$$1\% = 64 \div 10 = 6.4$$

$$2\% = 6.4 \times 2 = 12.8$$

$$64 + 12.8 = 76.8$$

Add to the original amount:

$$640 + 76.8 = 716.8$$

$$= 716.8 \text{ L}$$

A loss of 27% of 80cm

Find 27% of 80:

$$10\% = 80 \div 10 = 8$$

$$20\% = 8 \times 2 = 16$$

$$5\% = 8 \div 2 = 4$$

$$1\% = 8 \div 10 = 0.8$$

$$2\% = 0.8 \times 2 = 1.6$$

$$16 + 4 + 1.6 = 21.6$$

Subtract from the original amount:

$$80 - 21.6 = 58.4$$

$$= 58.4 \text{ cm}$$

## Questions

1. An increase of 20% of £450
2. A decrease of 11% on £65
3. A gain of 15% on 56kg
4. A loss of 23% on 76m
5. An increase of 6% on 75km
6. A loss of 35% on £15.60
7. A gain of 21% of £74
8. A decrease of 22% on £600
9. An increase of 17% on 922g
10. A decrease of 90% on 91mm
11. A gain of 27% on 805ml
12. A loss of 32% on £930

**Hints / Notes**

Find 1% by dividing by 10 twice or by dividing by 100 once! Find 5% by halving 10%. Check out subtracting decimal numbers if you can't remember how!

*Trouble with this? Jump to: **Percentages of a Quantity** (p. 105); **Dividing Decimal Numbers by Powers of Ten** (p. 154); **Subtracting Decimal Numbers Written Method** (p. 170)*

## 16.25 Percentage Increase and Decrease Problems

We can solve problems involving percentage increase and decrease.

### Example 16.45

Stefan bought 40 cakes for his party, the baker gave Stefan an extra 15% free. How many cakes did Stefan have for his party?

Find 15% of 40:

$$10\% = 40 \div 10 = 4$$

$$5\% = 4 \div 2 = 2$$

$$15\% = 4 + 2 = 6$$

Add to the original amount:

$$40 + 6 = 46$$

Stefan had 46 cakes.

Clare had 300 free text messages. Clare's phone company then gave her 12% extra. How many free text messages does Clare have now?

Find 12% of 300:

$$10\% = 300 \div 10 = 30$$

$$1\% = 30 \div 10 = 3$$

$$2\% = 3 \times 2 = 6$$

$$12\% = 30 + 6 = 36$$

Add to the original amount:

$$300 + 36 = 336$$

Clare has 336 free text messages.

Mark bought his car for £800. After a year it is worth 30% less. How much is the car worth now?

Find 30% of 800:

$$10\% = £800 \div 10 = £80$$

$$30\% = 80 \times 3 = £240$$

Subtract from the original amount:

$$800 - 240 = 560$$

The car is worth £560.

An antique vase cost £7500. The vase developed a crack. It is now worth 15% less. How much is it worth now?

Find 15% of 7500:

$$10\% = £7500 \div 10 = £750$$

$$5\% = £750 \div 2 = £375$$

$$15\% = 750 + 375 = £1125$$

Subtract from the original amount:

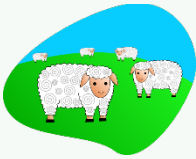
$$7500 - 1125 = 6375$$

The vase is worth £6375.

## Questions

1. Tom's pocket money was £16 per week. On his birthday, Tom had an increase of 30%. How much did Tom get on his birthday?

2. Jacob's farm had 450 sheep. In the spring, the sheep population increased by 12%. How many sheep were on the farm in spring?



3. An antique clock was worth £1600, the clock increased in value by 11%. How much is it worth now?

4. Toby picked 40 apples from his tree last year. This year Tony picked 15% more. How many apples did Toby pick this year?

5. A flat in Nottingham was priced at £70,000. Harry and Kim bought the flat for 11% less than the market value. How much did they pay for the flat?

6. Johanna walked 22 miles on Monday. On Tuesday, Johanna walked 24% more. How far did Johanna walk on Tuesday?



7. In a sale, the price of all wet suits was reduced by 20%. Ken bought a wet suit to swim in the sea. The original price was £65. How much did Ken pay in the sale?

8. Nadine's car was worth £8000. Nadine's friend Susanna also had a car. Susanna's car was worth 18% more. How much was Susanna's car worth?

9. Connor bought his first pair of glasses for £130. Recently, Connor bought some new glasses, they cost 10% less than before. How much did Connor's new glasses cost?



10. Abir's salary was £22,000. Her salary increased by 3%. How much does Abir earn now?

11. The Funtimes theme park charges £45 per visit. Students tickets cost 15% less. Clare is a student, how much does it cost Clare to visit the theme park?



12. Maria's dance classes cost £35 per week. If Maria pays for her classes in advance, the cost decreases by 12%. How much do the classes cost, if Maria pays in advance?

#### Hints / Notes

How do we increase or decrease by a percentage multiple times? To increase £40 by 10% 3 times, for example, we would increase £40 by 10%, the result is £44. We would increase £44 by 10%; the result is £48.40. We would then increase £48.40 by 10%; the result is £53.24!

*Trouble with this? Jump to: **Percentage Increase and Decrease** (p. 109); **Subtracting Decimal Numbers Written Method** (p. 170)*

## 16.26 Percentages of a Quantity Calculator Method

**Grade 3** To find percentages using a calculator, we can multiply by a single number to get the result. We change the percentage into a decimal, then we multiply by the decimal. For example, 10% as a decimal is 0.1, therefore to find 10% of an amount, we multiply it by 0.1. To find 30% of an amount, we multiply it by 0.3 and so on. This is called the multiplier method.

#### Key point

Change the percentage to a decimal, then multiply the amount by the decimal.

#### Example 16.46

Find these percentages.

60% of 980g

60% as a decimal is 0.6:

$$980 \times 0.6 = 588$$

$$= 588 \text{ g}$$

31% of 7.9m

31% as a decimal is 0.31:

$$7.9 \times 0.31 = 2.449$$

$$= 2.449 \text{ m}$$

7% of £35.20

7% as a decimal is 0.07:

$$35.2 \times 0.07 = 2.464$$

$$= \text{£}2.46$$

## Questions

1. 46% of £68.00
2. 15% of 4826g
3. 38% of £9.84
4. 27% of 66kg
5. 48% of £32.70
6. 95% of £58
7. 12% of 7.97m
8. 40% of £4.85
9. 3% of 98cm
10. 9% of 72.5L

### Hints / Notes

7% is 0.07, not 0.7, did you notice? To change a percentage to a decimal, divide by 100. Don't forget to round some of these answers to 2 decimal places, as they are in pounds and pence.

*Trouble with this? Jump to:* **Decimal Numbers and Percentages** (p. 92); **Percentages of a Quantity Non-Calculator Method** (p. 113); **Rounding to Decimal Places** (p. 5)

## 16.27 Percentage Increase Calculator Method

**Grade 3** To increase an amount by a percentage, we can multiply by a single number to get the result. We add 100% to the percentage, then we change the percentage to a decimal. We multiply the amount by the decimal.

To increase an amount by 20%, for example, we add 100% to 20%. 120% as a decimal is 1.2, we multiply the amount by 1.2.

### Key point

To increase an amount by a percentage, add 100% to the percentage then change it to a decimal. Multiply by the decimal.

### Example 16.47

Increase £30 by 40%.

Add 100% to 40%:

140% as a decimal is 1.4.

Multiply by 1.4:

$$30 \times 1.4 = 42$$

$$= \text{£}42.00$$

Increase 7.9m by 31%.

Add 100% to 31%:

131% as a decimal is 1.31.

Multiply by 1.31:

$$7.9 \times 1.31 = 10.349$$

$$= 10.349 \text{ m}$$

Find an increase of 7% on £35.20.

Add 100% to 7%:

107% as a decimal is 1.07.

Multiply by 1.07:

$$35.2 \times 1.07 = 37.664$$

Round to 2 decimal places for pounds and pence:

$$= \pounds 37.66$$

### Questions

1. Find an increase of 40% on £63.00.
2. Find an increase of 35% on 4.5L.
3. Increase 9.8cm by 32%.
4. Find an increase of 12% on 98 computer chips.
5. Find an increase of 58% on £362.
6. Increase £36.40 by 78%.
7. Increase £8.50 by 12%.
8. Find an increase of 5% on £850.
9. Find a gain of 21% on 50kg.
10. Find a gain of 2% on 55g.

### Hints / Notes

Change a percentage to a decimal by dividing by 100. A gain is an increase. We add 100% to the percentage to add on the original amount all at once!

*Trouble with this? Jump to: **Percentages of a Quantity – Calculator Method** (p. 113); **Rounding to Decimal Places** (p. 5)*

## 16.28 Percentage Decrease Calculator Method

**Grade 3** To decrease an amount by a percentage, we can multiply by a single number to get the result. We subtract the percentage from 100% then we change the remaining percentage to a decimal. We multiply the amount by the decimal.

To decrease an amount by 20% for example, we subtract 20% from 100%. We are left with 80%, 80% as a decimal is 0.8. We multiply the amount by 0.8.

### Key point

To decrease an amount by a percentage, subtract the percentage from 100% then change the remaining percentage to a decimal. Multiply by the decimal.

### Example 16.48

Decrease £48.00 by 10%.

Subtract 10% from 100%.

90% is left.

90% as a decimal is 0.9.

Decrease £66.30 by 35%.

Find a loss of 95% on £560.

$$48 \times 0.9 = 43.2$$

$$= \text{£}43.20$$

Subtract 35% from 100%.

65% is left.

65% as a decimal is 0.65.

$$66.3 \times 0.65 = 43.095$$

Write as pounds and pence:

$$= \text{£}43.10$$

Subtract 95% from 100%.

5% is left.

5% as a decimal is 0.05.

$$560 \times 0.05 = 28$$

$$= \text{£}28$$

### Questions

1. Find a decrease of 14% on £50.00.
2. Decrease £45 by 38%.
3. Find a loss of 30% on 240kg.
4. Find a loss of 11% on 3500 crops.
5. Decrease £242 by 27%.
6. Find a decrease of 88% on £605.
7. Find a decrease of 71% on 750ml.
8. Decrease 73ml by 92%.
9. Find a loss of 23% on 60kg.
10. Find a loss of 4% on 400 seeds.

### Hints / Notes

To change a percentage to a decimal divide by 100. A loss is a decrease.

*Trouble with this? Jump to: **Percentages of a Quantity – Calculator Method** (p. 113); **Percentage Increase – Calculator Method** (p. 114)*

## 16.29 Percentage Increase and Decrease Problems – Calculator Method

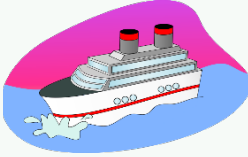
Use a calculator to solve the following percentage increase and decrease problems.

### Questions

1. There were 5000 poppies in a field. After a spell of good weather, the number of poppies increased by 17%. How many poppies are there now?



2. Jo's spending money was £5.50 per week. After her birthday, Jo's spending money increased by 15%. How much does Jo get now?
3. Henry booked a weekend cruise. The cruise costs £300, dinner costs 15% more. How much did Henry pay for the cruise, including dinner?



4. Lilly bought a tablet for £400 to take to College. One year later, the tablet had decreased in value by 30%. How much was the tablet worth one year later?
5. Karen bought 200 designer dresses to stock her new shop. In the second year, Karen bought 12% less stock. How many dresses did she buy in the second year?
6. Bran bought a guitar from the local music shop. The guitar was on sale at £240, but Bran bought it for 8% less. How much did Bran pay for the guitar?

## 16.30 Repeating Percentage Increase and Decrease Calculator Method

**Grade 3** To increase or decrease by a percentage, we multiply by a single number, called the multiplier. To increase or decrease by the percentage several times, we multiply several times! Multiplying several times is the same as raising the multiplier to a power.

For a repeating percentage increase, we add 100% to the percentage. For a repeating percentage decrease, we subtract the percentage from 100%. We change the percentage to a decimal. We raise the decimal to a power. We multiply by the new decimal.

For example, if we wished to increase 60kg by 20%, each year for 5 years, we could calculate:

$$60 \times 1.2 \times 1.2 \times 1.2 \times 1.2 \times 1.2.$$

This is the same as calculating:  $60 \times 1.2^5$ .

$$60 \times 1.2^5 = 149.2992\text{kg}.$$

### Key point

Multiply by a single number raised to a power, the power is the number of times to increase or decrease.

### Example 16.49

Find an increase of 15% on 300ml each year    Add 100% to 15%.

for 4 years.

115% as a decimal is 1.15.

Increase £150 by 6% each year for 2 years.

Find a decrease of 30% on 650g each year for 3 years.

Find a loss of 12% on £5000 each month for 4 months.

The increase is for 4 years, the power is 4:

$$300 \times 1.15^4 = 524.701875$$

$$= 524.70\text{ml (2.d.p)}$$

Add 100% to 6%.

106% as a decimal is 1.06.

The increase is for 2 years, the power is 2:

$$150.00 \times 1.06^2 = 168.54$$

$$= \text{£}168.54$$

Subtract 30% from 100%:

70% as a decimal is 0.70.

The decrease is for 3 years, the power is 3:

$$650 \times 0.70^3 = 222.95$$

$$= 222.95\text{g}$$

Subtract 12% from 100%:

88% as a decimal is 0.88.

The decrease is for 4 months, the power is 4:

$$5000 \times 0.88^4 = 2998.4768$$

$$= \text{£}2998.48$$

## Questions

Find the following increases or decreases.

1. An increase of 12% on £24 for 5 years.
2. A decrease of 17% on £65 for 2 years.
3. An increase of 30% on £24 for 2 years.
4. A decrease of 20% on £85 for 5 years.
5. A gain of 35% on 50kg for 3 years.
6. A loss of 5% on 40g for 4 years.
7. An increase of 5% on 70cm for 8 years.
8. A loss of 11% on £28 for 2 years.
9. A gain of 6% of £7.40 for 5 years.
10. A decrease of 7% on £380 for 6 years.
11. An increase of 15% on 92kg for 3 years.
12. A decrease of 90% on 60cm for 2 years.

Hints / Notes

*Trouble with this? Jump to: **Percentage Increase and Decrease** (p. 114)*

d.p.? It means decimal places! A repeating percentage increase, such as you get from the bank, is called compound interest.

## 16.31 Repeating Percentage Increase or Decrease Problems

We can solve problems involving repeating percentage increase or decrease.

### Example 16.50

Leo's salary was £12,000.

Each year Leo's salary increased by 13%. What is Leo's salary after 3 years?

Increase 12,000 by 13% for 3 years.

$$= 12000 \times 1.13^3$$

$$= 17314.764$$

Leo's salary is £17314.76

### Questions

1. A painting was worth £62,000. Each year, the value increased by 17%. How much was the painting worth after 6 years?

2. In a cake shop, you can buy any cake for £1.50. Each year, the cost of the cakes increases

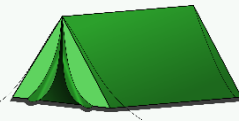


by 5%. How much will the cakes cost in 3 years' time?

3. Lynne's salary was £12,000. Each year Lynne's salary increased by 11%. What will Lynne's salary be after 4 years?

4. A small box of tea contains 500g. Each year the amount of tea put into the small box reduces by 6%. How much tea will be in the boxes in 2 years?

5. Last year Ren bought a tent for £400. Each month the tent's value decreased by 9%. After



4 months, how much was Ren's tent worth?

6. Isla's dog weighed 24kg. Each month Isla's dog lost 2% of her weight. How much does Isla's dog weigh after 6 months to the nearest kilogram?

7. The number of buses in Edinburgh was 121million. The number increased by 1.6%. If the number continues to increase by this amount each year, how many buses will there be in 5 years? Write your answer to the nearest whole number.

8. A pearl necklace was worth £20,000. Each year its value increases by 1.3%. What will the necklace be worth after 2 years?

9. Gita's walking distance was 16km. Each week she walked 8% further. After 3 weeks, how far is her walking distance to 1 decimal place?

10. A tennis ball bounces 140cm high when new. If, each week the ball loses 10% of its height.



How high will it bounce after 6 weeks?

11. The amount of toothpaste in a standard tube was 200ml. The amount of toothpaste has increased by approximately 12% each year. How much toothpaste is in a standard tube 3 years later, to the nearest millilitre?

12. Max bought a car for £800. Each year, the car's value decreased by 20%. How much is Max's car worth after 4 years?

#### Hints / Notes

For an increase, add 100 then write as a decimal. For a loss, subtract from 100 then write as a decimal. The number of years, months or weeks is the decimal's power.

*Trouble with this? Jump to: **Decimal Numbers and Percentages** (p. 92); **Percentage Increase and Decrease** (p. 114); **Rounding to Decimal Places** (p. 5)*

## 16.32 Percentage Profit and Percentage Loss

**Grade 3** If you buy something for a certain cost, then sell it again, you may want to know what kind of profit or loss you made. You may wish to know the *percentage profit or loss*, this is the profit or loss that you made, compared to its original value. The formulas for percentage profit and loss are shown below:

percentage profit =  $\text{profit} \div \text{original amount} \times 100$

percentage loss =  $\text{loss} \div \text{original amount} \times 100$

#### Key point

Divide the profit or loss by the original amount, then multiply by 100.

#### Example 16.51

Tia bought an old chair for £20, she painted the chair, then sold it for £32.

What was Tia's percentage profit?

Find the profit:

$$£32 - £20 = £12.$$

The original amount is £20.

$$12 \div 20 \times 100 = 60$$

The percentage profit is 60%.

Antonio bought leather shoes for £80 then sold them for £35.

What was Antonio's percentage loss to one decimal place?

Find the loss:

$$£80 - £35 = £45.$$

The original amount is £80.

$$45 \div 80 \times 100 = 56.25$$

The percentage loss is 56.3%.

## Questions

1. The Peacock family paid £400,000 for a detached house. Later, the family sold the house



for £460,000. What was the percentage profit?

2. Noah bought a plot of land for £26,000. Later, he sold it for £28,000. What was Noah's percentage profit?

3. An antique was purchased for £230, later it was sold for £278. What percentage profit is this?

4. Sally paid £21.00 for a new text book. After her college course, Sally sold the book for £15.50. What percentage loss was this?

5. A rucksack cost Edward £28. Edward sold the rucksack to his friend for £20. What percentage loss was this?

6. Johanne bought an antique lampstand for £55. A few months later Johanne sold it for £72. What percentage profit did Johanne make?

7. Nigel bought an old car for £780 two years ago. Nigel sold the car for £500. What was



the percentage loss?

8. Oscar bought a packet of flower seeds for £2.50. Oscar grew the flowers and then sold six bunches for £5 each. What was Oscar's percentage profit?

9. Sam bought a washing machine from his neighbour for £100. When Sam moved to a new house, he sold the washing machine for £175, what was Sam's percentage profit?

10. Jane bought a box of ten pens for £5, a box of ten pencils for £4.50 and a box of ten rulers for £6. Jane made gift boxes, each one containing a pen, a pencil and a ruler. Jane sold ten gift boxes for £3 each. What percentage profit is this?

## 16.33 Mixed Percentage Questions

**Grade 3** The tricky thing about percentages is answering all of the different types of questions!

## Questions

1. Sam scored 34 out of 60 for his History test, what percentage is this?
2. Nadine tried to buy 18 tickets for a coach trip, but she could only buy 12. What percentage did Nadine buy?
3. Veronika has flute and tuba lessons. 35% of Veronika's lessons are on the flute. What



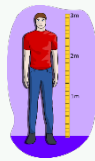
percentage are on the tuba?

4. Ann has tropical and cold-water fish. 42% of Ann's fish are tropical fish. What percentage are cold-water fish?
5. An antique tea set was on sale at £230. A customer bought the tea set for 75% of the price. How much did the customer pay for the tea set?
6. Steve receives £80.00 per week for working at the coffee shop. Steve spends 15% of his



salary on train tickets. How much does Steve spend on train tickets?

7. Sam the giant measured himself in the first year of High School, he was 2.5m high. When Sam measured himself again, he had grown 7% more. How tall was Sam the next time he



measured himself?

8. Clare bought a box of paints for £30 in December. In January, they cost 12% less. How much did the paints cost in January?
9. When the Happy Days Nursery opened, it cared for 30 children. Each year, the number of children increased by 5%. How many children were in the nursery 4 years later, to the nearest whole number?
10. The Kildare family bought a pretty terraced house for £80,000. Each year, the house increased in value by 3%. How much was the house worth after 5 years?
11. Leila bought a designer suitcase for £200. Leila sold the suitcase for £245. What was the percentage profit?
12. Tom purchased a computer game for £50. After completing the game, Tom sold it for £10.50. What was Tom's percentage loss?

### Hints / Notes

If you can't remember how to do the different questions, jump to the suggested topics!

*Trouble with this? Jump to:* **Fractions to Percentages** (p. 94); **Percentages** (p. 91); **Percentages of a Quantity – Calculator Method** (p. 113); **Percentage Increase and Decrease** (p. 114); **Percentage Profit and Percentage Loss** (p. 120)

## 16.34 Reverse Percentages

**Grade 3** Reverse percentages are calculated differently from other percentage questions. Reverse percentage questions ask: What was the original amount, before the change?

To find a reverse percentage, put the percentage increase (100% plus the percentage) or the percentage decrease (100% minus the percentage) equal to the amount. Find 1% of the amount by dividing by the percentage. Next, find 100%, the whole original amount, by multiplying by 100.

### Key point

Divide by the percentage increase or decrease, then multiply by 100.

### Example 16.52

After an increase in salary of 22%, Emma's pay was £380 per month. What was Emma's original salary?

Add 100% to 22%

$$100\% + 22\% = 122\%$$

Put the percentage equal to the amount:

$$122\% = \text{£}380$$

Divide by 122 to find 1%:

$$380 \div 122 = 3.11475\dots$$

Multiply by 100 to find 100%:

$$3.11475\dots \times 100 = 311.475\dots$$

Emma's original salary was £311.48.

A cereal box contains 756g, it has 12% extra free. How much would the cereal box contain originally?

Add 100% to 12%

$$100\% + 12\% = 112\%$$

Put the percentage equal to the amount:

$$112\% = 756$$

Divide by 112 to find 1%:

$$756 \div 112 = 6.75$$

Multiply by 100:

$$6.75 \times 100 = 675$$

The cereal box contained 675g originally.

After a loss due to sickness of 15% of the chickens, a farm had only 306 chickens left. How many chickens were there originally?

Subtract 15% from 100%

$$100\% - 15\% = 85\%$$

Put the percentage equal to the amount:

$$85\% = 306$$

Divide by 85 to find 1%:

$$306 \div 85 = 3.6$$

Multiply by 100:

$$3.6 \times 100 = 360$$

The number of chickens was 360.

## Questions

1. After an increase of 12%, Poppy's weight was 60kg, what was her weight before?
2. George counted 42 insects in a patch of ground, the number had increased by 20%. How



many did George count originally?

3. After an interest payment, Rob's bank balance increased by 5% to £320. How much did Rob have originally?
4. The highest history test mark this year was 72, this is 16% higher than last year. What was the highest mark last year?
5. A 90ml bottle has 15% extra free. How much is in a regular bottle?
6. After a loss of 12%, Elton's weight was 4000kg. What was Elton's original weight?



7. In a sale, all jackets had 17% off. Rita wanted the jacket that cost £45 in the sale. How much was the jacket originally?
8. After giving away 10% of his card collection, Simon was left with 180 cards, how many did Simon have before?
9. Roberto is sick, his temperature is currently 38.85°C. Roberto's temperature has gone up by 5%, what was his original temperature?
10. 20% of Lizzy's school money was lost on the way to school. Lizzy has only £2.00 left.



How much did Lizzy have before?

### Hints / Notes

Check your answer, should it be bigger or smaller than the original amount?

Trouble with this? Jump to: **Percentage Increase and Decrease** (p. 114)

## Chapter 17

# Simple Interest

### Grade 3

If you purchase a large item, such as a car or a television, you may not be able to pay for your purchase all at once. Instead of paying the total amount, you may pay part of the amount, then pay the rest of the money monthly or weekly. These payments are called instalments. If you choose to pay in this way, then you will be charged simple interest. This means that your purchase will cost more than the original purchase price, sometimes much more!

The formula for simple interest is:

#### Simple interest

Simple Interest = Amount  $\times$  Interest  $\times$  Number of years\*

The interest in the formula is written as a decimal number. If the interest is written as a percentage, then divide it by 100 to change it to a decimal number.

*\*This may be the number of months or weeks instead.*

#### Key point

Simple interest is the original amount multiplied by the interest (as a decimal) multiplied by the number of years, months or weeks.

#### Example 17.1

Heidi borrowed £200 over 3 months to buy a washing machine. The interest rate was 5% per month.

How much interest did Heidi pay?

Divide 5% by 100: 5% as a decimal is 0.05.

How much did the washing machine cost in total?

Use the formula:  $200 \times 0.05 \times 3 = 30$ .

Heidi paid £30 interest.

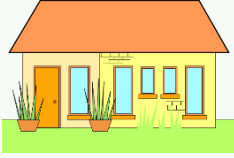
Add £30 to the original amount:  $200 + 30 = 230$ .

Heidi paid £230 in total for her washing machine.

#### Questions

Find the simple interest and the total amount for the following:

1. Martin borrowed £840 to buy a sofa, he borrowed the money over 4 years at 7% per year interest.
2. Jasmine borrowed £6800 for 5 years to buy a car. The interest rate was 7.5% per year.
3. Ian took out a loan to buy a house. He borrowed £50,000 over 15 years at 4% interest per



annum (per year).

4. Meghan took out a loan for £7740 over 10 months to buy a laptop. The rate was 3% interest per month.
5. Oliver borrowed £7000 over 2 years to pay for his wedding. The interest rate was 3.5% per year.
6. Regis took out a loan from the bank. He borrowed £950 for 4 months at 11% interest per month.

#### Hints / Notes

If the interest is a percentage, remember to change it to a decimal by dividing by 100.

*Trouble with this? Jump to: **Decimal Numbers and Percentages** (p. 92)*

## Chapter 18

# Ratio and Proportion

## 18.1 Understanding Ratio

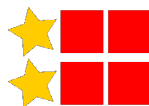
Grade 2

When we have different amounts of items, we can use ratios to compare them. To write a ratio we write the amounts side by side, with a colon (:) between them. For example, if we have a number of stars and a number of squares, then we can see how many stars we have, compared to the number of squares. When we have 1 star, we have 2 squares. When we have 2 stars, we have 4 squares. For every star there are 2 squares, the ratio of stars to squares is 1:2 (one to two).

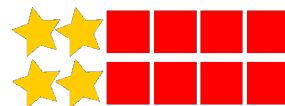
1 star, 2 squares



2 stars, 4 squares



4 stars, 8 squares



We could write the ratio of stars to squares as 4:8, then we could simplify this. Ratios can be simplified in the same way as fractions. If we divide both numbers in the ratio 4:8 by 4, then we get the ratio 1:2. To simplify ratios, each number is divided by the same amount.

### Key point

To simplify ratios, divide each number by the same amount.

### Example 18.1 — Simplify these ratios

$4 : 6 = 2 : 3$       4 and 6 can be divided by 2:  $4 \div 2 = 2$  and  $6 \div 2 = 3$ .

$15 : 18 = 5 : 6$       15 and 18 can be divided by 3:  $15 \div 3 = 5$  and  $18 \div 3 = 6$ .

### Questions A

Simplify these ratios.

1.  $12 : 28$

3.  $10 : 16$

5.  $26 : 54$

7.  $25 : 60$

9.  $72 : 45$

11.  $17 : 68$

13.  $15 : 20$

2.  $12 : 33$

4.  $11 : 55$

6.  $32 : 40$

8.  $36 : 18$

10.  $28 : 52$

12.  $24 : 36$

14.  $35 : 42$


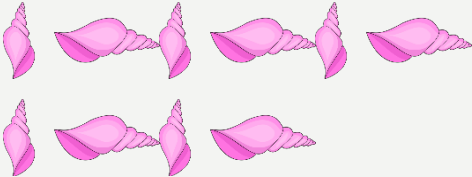
Hints / Notes

Trouble with this? Jump to: **Simplifying Fractions by Cancelling Down** (p. 74)

Divide each side by the same amount to make the numbers smaller!

If we are given a number of items, then we can compare the amounts and write down the ratio. We can then simplify the result.

**Example 18.2**  
Write down, then simplify the following ratio.


There are 5 butterflies and 10 shells, the ratio is 5:10.


Simplify, divide both numbers by 5:  $5 \div 5 = 1$  and  $10 \div 5 = 2$ .


The ratio of butterflies to shells is 1:2.


**Questions B**  
Write down the following ratios, simplifying if possible.


- Trains and cars.
 









- Swimmers and computer games.
 





- Cakes and weighing scales.
 

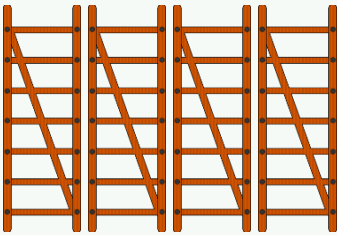








4. Coins and ladders.



## 18.2 Ratio and Fractions

Grade 2



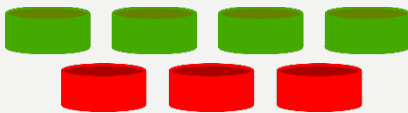
Writing a ratio is like writing a fraction. To write a ratio, we write the amounts side by side, with a colon between. To write a fraction, we write one amount above the other.

If we have 1 red counter and 3 blue counters then the *ratio* of red to blue counters is 1:3 (one to three). The *fraction* of red to blue counters is  $\frac{1}{3}$  (one over three). If we compare the number of red counters to the total number of counters, then the ratio would be 1 : 4 (one to four) and the fraction would be  $\frac{1}{4}$  (one over four).

### Key point

If the ratio is first amount : second amount then the fraction is  $\frac{\text{first amount}}{\text{second amount}}$ .

### Example 18.3



There are 3 red counters, 4 green counters and 7 counters in total.

What is the ratio of red counters to green counters? The ratio of red counters to green counters is 3:4.

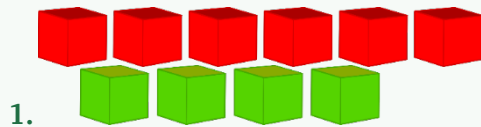
What is the fraction of red counters to green counters? The fraction of red counters to green counters is  $\frac{3}{4}$ .

The ratio of red counters to the total is 3:7.

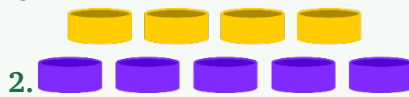
What is the ratio of red counters to the total number of counters? The fraction of red counters to the total is  $\frac{3}{7}$ .

What is the fraction of red counters to the total number of counters?

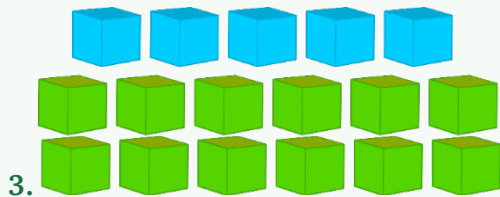
### Questions



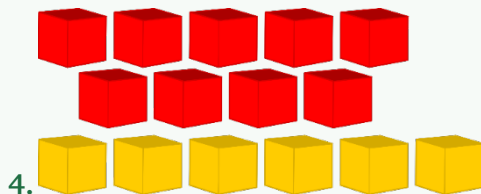
1. (a) What is the ratio of red to green cubes?, (b) What is the fraction of red cubes to the total number of cubes?, (c) What is the fraction of red cubes to green cubes?



2. (a) What is the ratio of yellow to purple counters?, (b) What is the ratio of yellow counters to the total number of counters?, (c) What is the fraction of yellow counters to purple counters?



3. (a) What is the fraction of blue to green cubes?, (b) What is the fraction of green cubes to the total number of cubes?, (c) What is the ratio of green cubes to blue cubes?



4. (a) What is the ratio of yellow cubes to the total number of cubes?, (b) What is the fraction of red cubes to the total number of cubes?, (c) What is the fraction of yellow cubes to red cubes?

#### Hints / Notes

Remember to write your ratios in the correct order. The ratio of blue to red is not the same as red to blue!

*Trouble with this? Jump to: **Fractions of a Whole** (p. 64)*

## 18.3 Ratio Problems

If we know one amount and the ratio of the amounts, then we can calculate the unknown quantity. We write the first amount in a table, then we work out the second amount by using the ratio. We

can add more amounts as we add more rows to the table. We continue in this way until we reach the quantity needed.

For example, if the ratio of bats to balls is 1:3, we know that for each bat there are 3 balls. We draw a table with 2 columns. In the first row we write 1 and 3, one bat, three balls. In the second row we write 2 and 6, two bats, six balls and so on. We continue this way until we have the correct number for the answer.

### Key point

Write the amounts in a table, using the ratio. Add amounts by adding new rows.

### Example 18.4

The ratio of dogs to cats in a rescue centre is 1:4. If there are 24 cats how many dogs are there?



There are 4 cats for each dog. Each time we add 1 dog, we add 4 cats. We continue adding rows until we have 24 cats.

Number of Dogs	Number of Cats
1	4
2	8
3	12
4	16
5	20
6	24

The table shows that when we have 24 cats we have 6 dogs. There are 6 dogs.

The ratio of Fairtrade bananas to ordinary bananas in the UK is 1:5. If a shop sells 42 bananas in total, how many were Fairtrade?

There are 5 ordinary bananas for every Fairtrade banana. This time we must work out the *total number* of bananas. Each time we add 1 Fairtrade banana, we add 5 ordinary bananas. We continue adding rows until we have 42 bananas in total.

Fairtrade	Ordinary	Total
1	5	6
2	10	12
3	15	18
4	20	24
5	25	30

Fairtrade	Ordinary	Total
6	30	36
7	35	42

The table shows that when the total is 42 we have 7 Fairtrade bananas. There are 7 Fairtrade bananas.

### Questions A

1. The ratio of boys to girls at a youth club is 3:1. If there are 18 boys how many girls are there?
2. The ratio of teas to coffees ordered in a cafe is 1:6. If 12 people ordered tea, how many people ordered coffee?



3. The ratio of adults to children at the nursery is 3:8. If there are 15 adults how many children do they care for?
4. Sabina's class voted for an end of term treat. They voted for either a picnic lunch or to go bowling in the ratio 3:5. If 12 students chose a picnic lunch, how many students chose to go bowling?
5. Adam and Alyson cut a rope in the ratio 4:5. If Alyson's rope is 15cm long, how long is Adam's rope?
6. The ratio of passengers to crew on a cruise ship is 11:2. If there are 65 people in total, how many people are part of the crew?



7. Clare and Amy split some cakes in the ratio 3:4. If Amy has 32 cakes, how many does Clare have?
8. Tom and Jake sent text messages in the ratio 1:3. If Tom and Jake sent 24 text messages in total, how many did Jake send?
9. Apple Fizz is made from apple juice and sparkling water in the ratio 1:5. How many litres of juice are there in 30 litres of Apple Fizz?

#### Hints / Notes

Writing out the numbers as a table helps at first!

*Trouble with this? Jump to: **Proportion** (p. 137)*

Instead of writing out tables to find answers to ratio questions, we can find the number that the ratio has been multiplied by. We can use this number to find the missing value.

**Key point**

Divide by the corresponding value in the ratio, then multiply by the other value.

**Example 18.5**

Charlie and Namar split their amusement park tickets in the ratio 2:5. If Namar has 15 tickets, how many does Charlie have?

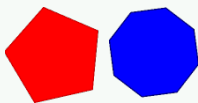
The number of tickets for Namar is 15, so  $2 : 5 = ? : 15$ . The 5 corresponds to the 15, divide 15 by 5:  $15 \div 5 = 3$ . The ratio has been multiplied by 3. Multiply 2 by 3:  $2 \times 3 = 6$ . Charlie has 6 tickets.

The ratio of main meals to puddings on a restaurant menu is 7:3. If there are 28 mains, how many puddings are there?

There are 28 mains, so  $7 : 3 = 28 : ?$ . The 7 corresponds to the 28, divide 28 by 7:  $28 \div 7 = 4$ . The ratio has been multiplied by 4. Multiply 3 by 4:  $3 \times 4 = 12$ . There are 12 puddings.

**Questions B**

1. Omar and Sunni have snack money in the ratio 3:4. If Omar has 90p how much does Sunni have?
2. The ratio of coach drivers to coaches in a travel company is 2:5. If the company has 22 coach drivers, how many coaches are there?
3. In a cookie recipe the ratio of sugar to flour is 1:4. If 50g of sugar was used, how much flour was used?
4. The ratio of Silver and Bronze medals won in the 2016 Rio Olympics between the US and the GB team were in the ratio 15:8. If the GB team won 40 medals, how many did the US team win? The total medal count achieved by China and Russia was in the ratio 5:4. If Russia won 56 how many did China win?
5. The ratio of first class to third class passengers that survived the disaster on the Titanic was 12:5. If 180 first class passengers were saved; how many third-class passengers were saved?
6. The ratio of the perimeters of a pentagon to an octagon is 5:8. If the perimeter of a pentagon is 45cm what is the perimeter of the octagon?

**Hints / Notes**

*Trouble with this? Jump to: **Equivalent Fractions** (p. 71)*

Finding equal ratios are like finding equivalent fractions!

## 18.4 Sharing in a Given Ratio

### Grade 3

We may have an amount or a number of items to be shared in a certain ratio. To do this, we add up the parts in the ratio to find the sum of the parts. We divide the amount by this sum to find the value of one part. We multiply each part by this value.

#### Key point

Add up the parts in the ratio, divide the amount by the sum to find the value. Multiply each part by the value.

#### Example 18.6

Abigail and Omar have 300 posters. They split the posters in the ratio 1:4. How many do they each have?

$1 + 4 = 5$ , there are 5 parts:  $300 \div 5 = 60$ . There are 60 posters in each part.



Abigail has 1 part:  $1 \times 60 = 60$ . Omar has 4 parts:  $4 \times 60 = 240$ . Abigail has 60 posters, Omar has 240.

Olga and Dimitri share £1.60 in the ratio 2:3. How much does Dimitri have?

$2 + 3 = 5$ , there are 5 parts:  $\pounds 1.60 \div 5 = \pounds 0.32$ . Each part has 32p.



Dimitri has 3 parts:  $3 \times 32 = 96$ . Dimitri has £0.96.

#### Questions

1. Tom and Jake share £400 in the ratio 1:4. How much do they each receive?
2. Clare and Amy split 140 marbles in the ratio 3:4. How many do they each have?
3. Adam and Jasmin cut a 300m rope in the ratio 1:5. How long is Jasmin's rope?
4. Carlos and Jose have 270g of chocolate. They break up the chocolate in the ratio 4:5. How many grams do they have each?

5. Nadine uses her computer for word processing and emails. Nadine spends 35 hours per week writing documents and emails in the ratio 3:4. How many hours does Nadine spend writing emails?



6. 540 votes were split between the Green Party and the Transport Party in the ratio 7:2. How many voters chose the Green Party?

7. Emily ran to keep up with her brother. They took 65 steps in total to reach the shops. If the ratio of Emily's steps to her brother's steps is 3:2, how many steps did Emily take?

8. The ratio of flowers to leaves on a plant is 13:20. If the total number of flowers and leaves is 198, how many flowers are there?

#### Hints / Notes

Add up the ratios, divide by the sum, then multiply by the new number.

Trouble with this? Jump to: **Understanding Ratio** (p. 127)

## 18.5 Ratio Questions with Algebra

### Grade 4

Some ratio problems are a little trickier as they involve using an unknown. We call the unknown  $x$ , then we use  $x$  to solve the problems.

If the ratio of two quantities is 2:3, for example, then the first quantity is  $2 \times$  something and the second quantity is  $3 \times$  something. The quantities can be written as  $2x$  and  $3x$ . We can find  $x$  from information in the question. Once we know  $x$ , we know the quantities.

#### Key point

If the ratio of two quantities is 2:3, for example, the quantities can be written as  $2x$  and  $3x$ .

#### Example 18.7

Christine and Daniel shared their postcards in the ratio 2:5. Daniel had 12 more postcards than Christine. How many postcards did each person have?

Use  $C$  for Christine and  $D$  for Daniel. Christine has 2 lots of something, Daniel has 5 lots:  $C = 2x$  and  $D = 5x$ . Daniel had 12 more postcards than Christine, write an equation:  $D = C + 12$ . Substitute for  $D$  and  $C$ :  $5x = 2x + 12$ . Solve the equation:  $3x = 12$ , so  $x = 4$ . Now we know

$x$ , we can find C and D:  $C = 2 \times 4 = 8$  and  $D = 5 \times 4 = 20$ . Christine has 8 postcards, Daniel has 20.

The length and width of a rectangle is in the ratio 3:1. The perimeter of the rectangle is 24cm. What is the length of the rectangle?

Use L for length and W for width. The length is 3 lots of something, the width is 1 lot of something:  $L = 3x$  and  $W = x$ . The perimeter of the rectangle is 24cm:  $2L + 2W = 24$ . Substitute for L and W:  $6x + 2x = 24$ . Solve the equation:  $8x = 24$ , so  $x = 3$ . Substitute back for L and W:  $L = 3 \times 3 = 9$  and  $W = 1 \times 3 = 3$ . The length is 9cm, the width is 3cm.

Ryan and Sanita share their photos in the ratio 2:7. Sanita then gives Ryan 15 of her photos. The ratio is now 1:1. How many photos does Ryan now have?

Use R for Ryan and S for Sanita:  $R = 2x$  and  $S = 7x$ . Sanita then gives Ryan 15 of her photos. Ryan now has 15 more, Sanita now has 15 less:  $R = 2x + 15$  and  $S = 7x - 15$ . The new ratio is 1:1 so they now have the same amount:  $2x + 15 = 7x - 15$ . Solve the equation:  $30 = 5x$ , so  $x = 6$ . Substitute back for R or S:  $R = 2 \times 6 + 15 = 27$ . They both have 27 photos.

## Questions

1. Simone and Tobias shared their plant seeds in the ratio 4:5. Tobias had four more plant seeds than Simone. How many seeds did Simone have?
2. The length and width of a rectangle is in the ratio 3:2. The perimeter of the rectangle is 20cm. What is the length of the rectangle?
3. Mika and Kira shared their sweets in the ratio 2:3. Kira had 5 more sweets than Mika. How many sweets did each person have?



4. The height and diameter of a cylinder is in the ratio 11:5. The height is 18cm taller than the diameter. What is the *radius* of the cylinder?
5. A shopping bag contains onions and potatoes in the ratio 2:11. There are 27 more potatoes than onions. How many potatoes are there?
6. An isosceles triangle has a height and a base length in the ratio 5:2. The base is 6cm shorter than the height. What length is the base?



7. Michael and Ross share their spending money in the ratio 3:7. Ross has £18 more than Michael. How much does Michael have?

8. Jules and Kate share some conkers in the ratio 3:5. Kate then gives Jules 5 conkers. The ratio is now 1:1. How many conkers does Kate now have?

#### Hints / Notes

Make sure that you can share in a given ratio really well before attempting these! Look up solving equations in the Algebra book and Perimeter of a rectangle in the Shapes book.

Trouble with this? Jump to: **Sharing in a Given Ratio** (p. 134)

## 18.6 Proportion

### Grade 2

When quantities are in proportion, it means that if one quantity changes then the other quantity changes in the same way. At a garden party for example, the number of party hats and balloons are in proportion. If we double the number of party hats, then we double the number of balloons. If we multiply the number of party hats by five, then we multiply the number of balloons by five.

#### Key point

If we multiply one amount, then we multiply the other amount by the same number.

#### Example 18.8

One bag of popcorn costs £1.20. How much will three bags of popcorn cost?

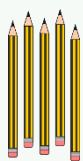
1 bag = £1.20. To get 3 bags we multiply the number of bags by 3, therefore we multiply the cost by 3:  $£1.20 \times 3 = £3.60$ . Three bags of popcorn will cost £3.60.

Three cans of lemonade cost £2.56. How much will twelve cans of lemonade cost?

3 cans = £2.56. To get 12 cans, we multiply 3 cans by 4. Therefore, we multiply the cost by 4:  $2.56 \times 4 = 10.24$ . Twelve cans of lemonade will cost £10.24.

### Questions

- 1 chocolate bar costs 85p. How much will 4 bars cost?
- 5 pencils cost £1.20. How much will 15 pencils cost?



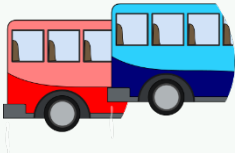
- 4 knobs of butter weigh 110g. How much will 20 knobs of butter weigh?

4. 9 sandwiches are stacked on top of each other, they are 12cm high. How high will 27 sandwiches be?

5. 5 candles cost £4.00. How much will forty candles cost?

6. If 1 ferry boat to the Isle of Wight can hold 110 vehicles, how many vehicles can 7 ferry boats hold?

7. 8 people can fit into a minibus. If 48 people are travelling, how many minibuses will be needed?



8. A lift can carry 12 people, if 60 people are waiting to use the lift, how many times will the lift be needed?

9. If 3 smoothies cost £7.60, how much will 18 smoothies cost?

10. 14 snails were found in 20 potted plants. How many snails would be found in 100 potted plants?



11. Sally can fit 6 of her folders in 1 box. If Sally has 5 boxes, how many folders can she fit in?

12. 12 mixed spanners cost £180. How much will 36 mixed spanners cost?

#### Hints / Notes

A quick way of multiplying by 5 is to multiply by ten then halve the answer. A quick way of multiplying by 4, is to double the number twice!

## 18.7 Proportion Unitary Method

### Grade 3

When quantities are in proportion, it means that if one quantity changes then the other quantity changes in the same way. Sometimes we are given items that are in proportion, but we cannot simply multiply the amount to get the answer. Instead, we divide the quantities to find the value of one item, then we can multiply to find the answer. This method is called the unitary method.

#### Key point

Divide by the number of items to find the value of one item, then multiply by this value.

#### Example 18.9

If 8 small pizzas cost £40. How much will 10 pizzas cost?

8 pizzas = £40. Find the cost of one pizza, divide by 8:  $40 \div 8 = 5$ . 1 pizza costs £5.00. Find the cost of 10 pizzas:  $5 \times 10 = 50$ . 10 pizzas cost £50.

If 6 apples weigh 900g. How much will 7 apples weigh?

6 apples = 900g. Find the weight of one apple, divide by 6:  $900 \div 6 = 150$ . 1 apple weighs 150g. Find the weight of 7 apples:  $150 \times 7 = 1050$ . 7 apples weigh 1050g or 1.05kg.

## Questions

- 6 sofa cushions cost £282. How much will eight sofa cushions cost?
- 6 litres of paint cover 38 square metres of a room. How many square metres can Gina paint with 15 litres?
- 9 students went to the cinema, they paid £76.50 in total. How much would it cost for 10 students?
- A recipe to make 12 shortbread biscuits uses 200g of butter and 300g of flour. How much butter and flour would be needed to make 8 biscuits?
- 3 shopping bags can hold 15 cakes. Rob has 5 shopping bags, how many cakes can he carry?



- It costs £24 for 3 people to visit the wildlife park. How much will it cost for 5 people?
- 24 bulbs weigh 440g. How much will 30 bulbs weigh?
- A brownie recipe for 12 people uses 180g butter, 80g flour and 260g sugar. How much butter and flour are needed to make brownies for 8 people?
- 150 screws weigh 400g. How many screws would weigh 1kg?
- Tina counted the words on the first 3 pages of her story. Tina had written 330 words. Tina must write 1650 words for her homework assignment, how many more pages should Tina write?

### Hints / Notes

The unitary method means finding one of something first.

*Trouble with this? Jump to: **Proportion** (p. 137)*

## 18.8 Exchange Rates

Grade 3

If you travel to a different country for a holiday, you may want to change your money from UK pounds to the currency of the country that you are visiting. The amount you get depends on the exchange rate. The exchange rate shows the price of one currency compared to another. The exchange rate changes daily.

Exchange rates show amounts of money that are in proportion. We find the amount of money in a new currency by multiplying or dividing by the exchange rate. If the exchange rate shows *one* of the currency we need to change into we multiply, if not, we divide.

### Key point

If the exchange rate shows *one* of the currency needed we multiply, if not, we divide.

### Example 18.10

The exchange rate is £1 = \$1.50 (one pound is equal to one dollar and fifty cents). Tom changed £70.00 into dollars, how many dollars did Tom receive?

We need £70. We need to multiply £1 by 70 to get £70, therefore we multiply \$1.50 by 70 to get dollars:  $1.50 \times 70 = 105$ . Tom will receive \$105.00.

The exchange rate is €1 = £0.80 (1 Euro is equal to eighty pence). If Belle changes £16.00 into Euros, how many Euros will she get?

We need £16.00. We need to know how many £0.80's there are in £16.00, this will tell us how many Euros we have. Divide £16.00 by £0.80:  $16.00 \div 0.80 = 20$ . Belle gets €20.00.

The exchange rate is 1 HRK = £0.12 (HRK - Croatian Kuna). Leon changed 250 HRK into pounds, how many pounds did Leon receive?

We need 250 HRK. We need to multiply 1 HRK by 250 to get 250 HRK, therefore we multiply £0.12 by 250 to get pounds:  $0.12 \times 250 = 30$ . Leon receives £30.

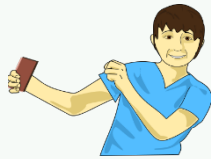
### Questions A

1. The exchange rate was £1 = \$1.50. Sally changed £80.00 into dollars for a school trip to New York. How many dollars did Sally get?



2. The exchange rate was €1 = £0.64. Arthur planned to take the train from Paris to Waterloo for a long weekend. Arthur changed €150.00 into pounds, how much did he have to spend in the UK?

3. The exchange rate was  $\text{£}1 = \$1.60$  NZD (NZD - New Zealand Dollars). Hal changed  $\text{£}50$  to pay for his bus trip to Rotorua, how many New Zealand Dollars did Hal receive?
4. The exchange rate was  $1 \text{ HRK} = \text{£}0.12$  (HRK - Croatian Kuna). How much did Lizzy receive when she changed 140 HRK after returning from Dubrovnik?
5. The exchange rate was  $\text{€}1 = \text{£}0.64$ . Kerry visited Rome for a holiday. On her return, Kerry changed  $\text{€}17.00$  into pounds. How many pounds did she get?
6. The exchange rate was  $\text{€}1 = \text{£}0.75$ . Daniel changed  $\text{£}108.00$  into Euros before going on an exchange trip to Berlin. How many Euros did he get?



7. The exchange rate was  $\text{€}1 = \text{£}0.65$ . Leon changed  $\text{£}140$  into Euros to take a trip to The Netherlands. How many Euros and Cents did Leon get?
8. The exchange rate was  $\text{£}1 = \text{¥}195$  (195 Japanese Yen). Mira changed  $\text{¥}12675$  to pounds before visiting the UK. How many pounds did Mira get?
9. The exchange rate was  $\text{£}1.00 = \text{€}1.12$ . Kerry changed  $\text{€}174$  into pounds before visiting her cousins in Wales. How much did she receive?
10. The exchange rate was  $\text{£}1.00 = \text{€}1.15$ . Max received  $\text{€}25.00$  from his Spanish family for his birthday. Max changed his birthday money into pounds. How much did Max get?

#### Hints / Notes

If the exchange rate shows one of the number you need, then multiply, if it doesn't, then divide first. Think, should the answer be bigger or smaller than before?

*Trouble with this? Jump to: **Proportion** (p. 137)*

We can compare prices of items in different countries by changing one of the prices, using the exchange rate. When the prices are in the same currency, we can compare them.

#### Example 18.11

The exchange rate is  $\text{€}1 = \text{£}0.64$ . In Barcelona, a sandwich and a drink costs  $\text{€}17.50$ . In London, the same meal costs  $\text{£}15.40$ . In which city is the meal cheaper?

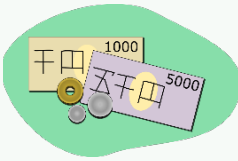
Convert  $\text{€}17.50$  into pounds:  $17.50 \times 0.64 = \text{£}11.20$ . In Barcelona the meal would cost  $\text{£}11.20$ . In London the meal costs  $\text{£}15.40$ . The meal is cheaper in Barcelona, Spain.

The exchange rate is  $\text{£}1 = \$1.50$ . Two cinema tickets cost  $\$40$  in Boston. The tickets cost  $\text{£}34.50$  in York. In which city are the tickets cheaper?

Convert £34.50 into dollars:  $34.5 \times 1.50 = \$51.75$ . In York the tickets would cost \$51.75. The tickets cost \$40 in Boston. The tickets are cheaper in Boston, US.

### Questions B

1. The exchange rate is  $\text{€}1 = \text{£}0.65$ . In Dublin, a pizza and a drink costs  $\text{€}13.00$ . In Liverpool, the same meal costs  $\text{£}13.40$ . In which city is the meal cheaper?
2. The exchange rate is  $\text{£}1 = \$1.50$ . A pair of shoes costs \$42.30 in Orlando. The same pair costs  $\text{£}35.60$  in Birmingham. Where should you buy the shoes?
3. The exchange rate is  $\text{£}1 = \text{¥}195$  (195 Yen). A jacket costs  $\text{¥}4500$  in Tokyo and  $\text{£}45$  in Manchester. In which city is the jacket cheaper?



4. The exchange rate is  $\text{£}1 = \$1.60$  (NZD – New Zealand Dollars). If a bottle of sparkling water costs 88p in Bristol and \$1.10 in Christchurch, in which city is the drink cheaper?
5. The exchange rate is  $\text{£}1 = 75.70$  RUB (Russian Rubles). A computer costs 55000 RUB in Moscow and the same computer costs  $\text{£}750$  in Exeter. In which city should you buy the computer?
6. The exchange rate is  $\text{£}1 = 0.043$  EGP (Egyptian pounds). If a museum ticket costs 0.5 EGP in Cairo and  $\text{£}9.50$  in Glasgow, what is the difference between the prices in each city?

## Chapter 19

# Using a Calculator

### Grade 3

Sometimes when you enter a calculation into a calculator you may get a different answer to someone else, why is that? Could it be a problem with the way that you entered the numbers? The examples show how to enter calculations to avoid errors.

#### Example 19.1

(a)  $\frac{32.4 \times 17.5}{12.3 - 4.9}$

(b)  $(32.4 + 17.5) \div 3.56$

(c)  $\frac{4.5^3}{3.2 - 0.3^2}$

(a) Using the fraction button, enter the calculation as it is shown:  $\frac{\square}{\square}$ . Press the S $\leftrightarrow$ D button to display the answer as a decimal, then round to 2 decimal places: = 76.62 (2 d.p.).

(b) Enter the calculation exactly how it is written, using the brackets:  $(32.4 + 17.5) \div 3.56$ . Round the answer to 2 decimal places: = 14.02 (2 d.p.).

(c) Enter the fraction using the fraction button. Use the  $x^3$  button to enter the number on the top line and the  $x^2$  button to enter the number on the bottom line. Press the S $\leftrightarrow$ D button to display the answer as a decimal, then round to 2 decimal places: = 29.3 (2 d.p.).

### Questions

Use your calculator to work out each of these, rounding your answer to 2 decimal places.

1.  $\frac{32.4 \times 17.5}{12.3 - 4.9}$

3.  $\frac{42.3 + 8.2}{1.5 - 0.3}$

5.  $\frac{12.5 - 0.2}{8.3 \times 4.1}$

7.  $(32.4 + 17.5) \div 3.56$

9.  $(48.4 - 23.2) \div (3.47 + 2.1)$

2.  $\frac{4.03 \times 7.2}{6.5^2 - 0.8}$

4.  $\frac{1.5^2 - 0.8}{4.76 - 1.6^3}$

6.  $\frac{0.3 \times 7.2}{91.4 + 2.05^2}$

8.  $(91.4 + 2.05^2) \div 0.12$

10.  $(3.24 - 0.102) \div (5.1 - 9.1)^2$

#### Hints / Notes

Read the numbers carefully — most mistakes are made by entering the wrong num-

*Trouble with this? Jump to: **BIDMAS** (p. 60)*

bers. Remember to press [=] after entering the fraction.

## Chapter 20

# Upper and Lower Bounds

### Grade 4

When we measure something, the measurement will never be exact, it will have an error, no matter how accurate the measuring equipment. The accuracy of a measurement is the nearest unit that you choose to measure to. To measure the height of a mountain, for example, you may measure to the nearest 100m, but to measure the width of a human hair, the nearest 0.01mm may be more appropriate. The error in each measurement is the accuracy divided by 2.

If we measure the length of a book, for example, it may be 23cm to the nearest 1cm. The accuracy is 1cm. The error is  $1\text{cm} \div 2 = 0.5\text{cm}$ . The length of the book could actually be as large as 23.5cm or as small as 22.5cm.

The largest that a measurement can be is called the *upper bound*, the smallest that a measurement can be is called the *lower bound*. The upper bound is the original measurement plus the error, the lower bound is the original measurement minus the error.

#### Key point

Error = Accuracy  $\div$  2.

Upper Bound = Measurement + Error.

Lower Bound = Measurement – Error.

The lower bound is the measurement minus half the accuracy, the upper bound is the measurement plus half the accuracy.


#### Example 20.1

Fill in the table for these measurements.

Measurement	Error	Upper Bound	Lower Bound
An egg cup weighs 65g to the nearest gram.	0.5g	$65 + 0.5 = 65.5\text{g}$	$65 - 0.5 = 64.5\text{g}$
A dog weighs 80kg to the nearest 10kg.	5kg	$80 + 5 = 85\text{kg}$	$80 - 5 = 75\text{kg}$
A teaspoon measures 13.7cm to the nearest 0.1cm.	0.05cm	$13.7 + 0.05 = 13.75\text{cm}$	$13.7 - 0.05 = 13.65\text{cm}$

## Questions

Write down the error, the upper bound and the lower bound for these measurements.

1. A pen weighs 23g to the nearest gram.
2. A person weighs 60kg to the nearest 10kg.
3. A pencil sharpener measures 2.5cm to the nearest 0.1cm. 
4. An elephant weighs 4800kg to the nearest 100kg.
5. A sandwich weighs 65g to the nearest 5g.
6. The distance from Cambridge to Bristol is 250 miles to the nearest 50 miles.
7. A pigeon measures 58cm to the nearest 2cm.
8. A parcel weighs 740g to the nearest 20g.

### Hints / Notes

Read the accuracy, calculate the error, then you can work out the bounds!

## 20.1 Writing Bounds as Inequalities

We can write measurements as inequalities. We find the upper and lower bounds, then we write the lower bound to the left and the upper bound to the right of the measurement. The inequality is written in the format  $LB \leq x < UB$ . The  $x$  is a letter representing the measurement of the object, like  $p$  for person.

### Key point

Write inequalities of bounds in the format  $LB \leq x < UB$ .

### Example 20.2

Write these upper and lower bounds as an inequality.

Measurement	Error	Lower Bound	Upper Bound	$LB \leq x < UB$
A trunk $t$ weighs 90kg to the nearest 10kg.	5kg	85kg	95kg	$85 \leq t < 95$
A tin of beans $b$ weighs 25.8g to the nearest 0.1g.	0.05g	25.75g	25.85g	$25.75 \leq b < 25.85$
A person $p$ weighs 92.63kg to the nearest 0.01g.	0.005g	92.625g	92.635g	$92.625 \leq p < 92.635$

## Questions

Write these upper and lower bounds as an inequality.

1. A tin of baby food,  $b$ , weighs 27.2g to the nearest 0.1g.
2. A person,  $p$ , weighs 92.8kg to the nearest 0.2kg.
3. A serving spoon,  $s$ , measures 40cm to the nearest 10cm.
4. A baby hippo,  $h$ , weighs 420kg to the nearest 20kg.
5. A running track,  $t$ , measures 800m to the nearest 100m.
6. A dessert fork,  $f$ , measures 25cm to the nearest 5cm.
7. A toy car,  $c$ , weighs 20kg to the nearest 10kg.
8. A bicycle,  $b$ , weighs 7.8kg to the nearest 0.1kg.

#### Hints / Notes

Why does the inequality have  $\leq$  on the left and only  $<$  on the right? Because, say, if we have a measurement, 90cm to the nearest 10cm then the inequality is  $85 \leq 90 < 95$ . 85 is the smallest number that rounds up to 90, the largest number that rounds down to 90 is less than 95 (not equal to).

Chapter 21

# Help Section

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## Chapter 22

# Place Value

The place value is the value that digits have depending on their position in the number. For example, in the number 8326 the 8 has the value 8000 and the 3 has the value 300.

## 22.1 Place Value of Whole Numbers Grade 2

**Grade 2** The Units column is on the right of the number. As we move left the numbers become ten times bigger.

### Example 22.1

Write the value of each digit in the following numbers.

**2563** — write the digits in their columns:

Thousands	Hundreds	Tens	Units
2	5	6	3

The value of the 2 is 2000. The value of the 5 is 500. The value of the 6 is 60. The value of the 3 is 3.

**1095** — write the digits in their columns:

Thousands	Hundreds	Tens	Units
1	0	9	5

The value of the 1 is 1000. The value of the 9 is 90. The value of the 5 is 5.

### Questions A

Write the value of each digit for the following numbers.

- 3491
- 2900
- 9015
- 9201
- 631
- 2005
- 7830
- 988

## 22.2 Place Value of Decimal Numbers Grade 2

**Grade 2** The digits to the right of the decimal point are parts of a whole number. As we move to the right the numbers become ten times smaller. The Tenths column is to the right of the decimal point. The Hundredths column is to the right of the Tenths column and so on.

**Example 22.2**

**42.87** — write the digits in their columns:

Tens	Units	.	Tenths	Hundredths
4	2	.	8	7

The value of the 4 is 40. The value of the 2 is 2. The value of the 8 is 0.8 or 8 tenths. The value of the 7 is 0.07 or 7 hundredths.

**3.209** — write the digits in their columns:

Units	.	Tenths	Hun- dredths	Thou- sandths
3	.	2	0	9

The value of the 3 is 3. The value of the 2 is 0.2 or 2 tenths. The value of the 9 is 0.009 or 9 thousandths.

**Questions B**

Write the value of each digit for the following numbers.

1. 1.452
2. 56.325
3. 0.514
4. 324.56
5. 25.312
6. 0.804
7. 603.09
8. 30.015

**Note**

Decimal digits can be very small. For example, if you were given 0.5 of a bar of chocolate, the bar would be split into tenths (10 pieces) and you have 5 tenths, half of the bar. However, if you were given 0.05 of the chocolate, the bar is now split into hundredths (100 pieces) and you would have only 5 of them! A very small amount!

**22.3 Multiplying Whole Numbers by Powers of Ten Grade 2**

**Grade 2** To multiply a number by ten, we move each digit one place to the left in the place value table.

**Example 22.3**

$72 \times 10 =$  We write 72 in the place value table. The 7 is in the Tens column and the 2 is in the Units column:

Hundreds	Tens	Units	.	Tenths
	7	2	.	

Hundreds	Tens	Units	.	Tenths
7	2		.	
	←	←		

The 7 moves one place to the left, the 2 moves one place to the left. There are zero units so the number becomes 720.

$$72 \times 10 = 720$$

$891 \times 10 =$  We write 891 in the place value table:

Thousands	Hundreds	Tens	Units
8	9	1	
	←	←	←
8	9	1	

Each digit moves one place to the left. The number becomes 8910.

$$891 \times 10 = 8910$$

### Questions

- |   |   |
|---|---|
| <p>1. <math>8 \times 10</math></p> <p>3. <math>892 \times 10</math></p> <p>5. <math>324 \times 10</math></p> <p>7. <math>25 \times 10</math></p> <p>9. <math>816 \times 10</math></p> <p>11. <math>106 \times 10</math></p> <p>13. <math>1708 \times 10</math></p> <p>15. <math>2 \times 100</math></p> <p>17. <math>54 \times 100</math></p> <p>19. <math>816 \times 100</math></p> <p>21. <math>1064 \times 100</math></p> <p>23. <math>4 \times 1000</math></p> <p>25. <math>15 \times 1000</math></p> <p>27. <math>855 \times 1000</math></p> | <p>2. <math>7 \times 10</math></p> <p>4. <math>96 \times 10</math></p> <p>6. <math>627 \times 10</math></p> <p>8. <math>3 \times 10</math></p> <p>10. <math>20 \times 10</math></p> <p>12. <math>980 \times 10</math></p> <p>14. <math>5431 \times 10</math></p> <p>16. <math>6 \times 100</math></p> <p>18. <math>32 \times 100</math></p> <p>20. <math>207 \times 100</math></p> <p>22. <math>9800 \times 100</math></p> <p>24. <math>8 \times 1000</math></p> <p>26. <math>39 \times 1000</math></p> <p>28. <math>227 \times 1000</math></p> |
|---|---|

To multiply a number by 100, we multiply by 10, then multiply by 10 again. To multiply by 1000, we multiply by 10 three times.

#### Hints / Notes

To multiply a whole number by ten, we add a zero to the right of the number.

*Trouble with this? Jump to: **Place Value** (p. 149)*

## 22.4 Dividing Whole Numbers by Powers of Ten Grade 2

**Grade 2** To divide a number by ten, we move each digit one place to the right in the place value table.

**Example 22.4**

$430 \div 10 =$  We write 430 in the place value table. The 4 is in the Hundreds column, the 3 is in the Tens column and the 0 is in the Units column:

Hundreds	Tens	Units	.	Tenths
4	3	0	.	
→	→	→	.	
	4	3	.	0

The 4 moves one place to the right, the 3 moves one place into the Units column and the 0 moves one place to the right over the decimal point into the Tenths column. 43.0 is the same as 43.

$$430 \div 10 = 43$$

$80 \div 10 =$  We write 80 in the place value table. Each digit moves one place to the right. 8.0 is the same as 8.

$$80 \div 10 = 8$$

$65 \div 10 =$  We write 65 in the place value table. The digits move one place to the right, some crossing over the decimal point. The 6 becomes 6 units and the 5 becomes 5 tenths. The number becomes 6.5.

$$65 \div 10 = 6.5$$

**Questions**

- |   |   |
|---|---|
| <p>1. <math>4850 \div 10</math><br/>           3. <math>800 \div 10</math><br/>           5. <math>685 \div 10</math><br/>           7. <math>420 \div 10</math><br/>           9. <math>30 \div 10</math><br/>           11. <math>25 \div 10</math><br/>           13. <math>89 \div 10</math><br/>           15. <math>7 \div 10</math><br/>           17. <math>2756 \div 100</math><br/>           19. <math>892 \div 100</math><br/>           21. <math>76 \div 100</math><br/>           23. <math>20 \div 100</math><br/>           25. <math>70 \div 100</math><br/>           27. <math>8912 \div 1000</math><br/>           29. <math>764 \div 1000</math><br/>           31. <math>25 \div 1000</math></p> | <p>2. <math>2830 \div 10</math><br/>           4. <math>450 \div 10</math><br/>           6. <math>218 \div 10</math><br/>           8. <math>451 \div 10</math><br/>           10. <math>70 \div 10</math><br/>           12. <math>62 \div 10</math><br/>           14. <math>17 \div 10</math><br/>           16. <math>9 \div 10</math><br/>           18. <math>6254 \div 100</math><br/>           20. <math>173 \div 100</math><br/>           22. <math>98 \div 100</math><br/>           24. <math>9 \div 100</math><br/>           26. <math>6 \div 100</math><br/>           28. <math>1073 \div 1000</math><br/>           30. <math>908 \div 1000</math><br/>           32. <math>81 \div 100</math></p> |
|---|---|

To divide a number by 100, we divide by 10, then divide by 10 again. To divide by 1000, we divide by 10 three times.

**Hints / Notes**

*Trouble with this? Jump to: **Place Value** (p. 149)*

If a whole number ends in zero, then to divide it by ten we simply remove the zero!

## 22.5 Multiplying Decimal Numbers by Powers of Ten Grade 2

**Grade 2** We multiply decimal numbers in the same way as whole numbers. To multiply a number by ten, we move each digit one place to the left in the place value table.

### Example 22.5

$2.3 \times 10 =$  We write 2.3 in the place value table. The 2 in the Units column and the 3 in the Tenths column.

Tens	Units	.	Tenths	Hundredths
	2	.	3	
	←	.	←	
2	3	.		

The 2 moves one place to the left, the 3 moves one place to the left, over the decimal point, into the Units column. The number becomes 23.

$$2.3 \times 10 = 23$$

$0.15 \times 10 =$  We write 0.15 in the place value table. The 0 is in the Units column, the 1 is in the Tenths column and the 5 is in the Hundredths column.

Tens	Units	.	Tenths	Hundredths
	0	.	1	5
	←	.	←	←
0	1	.	5	

The digits all move one place to the left, some crossing over the decimal point. The number becomes 1.5.

$$0.15 \times 10 = 1.5$$

### Questions

- |                        |                        |
|------------------------|------------------------|
| 1. $0.5 \times 10$     | 2. $0.3 \times 10$     |
| 3. $6.5 \times 10$     | 4. $9.3 \times 10$     |
| 5. $4.25 \times 10$    | 6. $6.11 \times 10$    |
| 7. $3.24 \times 10$    | 8. $37.8 \times 10$    |
| 9. $92.9 \times 10$    | 10. $7.55 \times 10$   |
| 11. $0.14 \times 10$   | 12. $0.88 \times 10$   |
| 13. $7.087 \times 10$  | 14. $60.31 \times 10$  |
| 15. $4.506 \times 10$  | 16. $16.03 \times 10$  |
| 17. $0.87 \times 100$  | 18. $0.22 \times 100$  |
| 19. $0.578 \times 100$ | 20. $0.195 \times 100$ |

21.  $7.037 \times 100$
23.  $42.506 \times 100$
25.  $0.8237 \times 1000$
27.  $0.543 \times 1000$
29.  $7.07 \times 1000$
31.  $85.506 \times 1000$

22.  $6.891 \times 100$
24.  $16.03 \times 100$
26.  $0.2129 \times 1000$
28.  $0.215 \times 1000$
30.  $6.41 \times 1000$
32.  $0.0163 \times 1000$

**Hints / Notes**

If you are having trouble remembering which way the numbers move, try to make a rhyme out of it. Multiply, Divide is Left and Right!

*Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)*

## 22.6 Dividing Decimal Numbers by Powers of Ten Grade 2

**Grade 2** We divide decimal numbers in the same way as whole numbers. To divide a number by ten, we move the number one place to the right in the place value table.

**Example 22.6**

$13.5 \div 10 =$  We write 13.5 in the place value table. The 1 is in the Tens column, the 3 is in the Units column and the 5 is in the Tenths column.

Tens	Units	.	Tenths	Hundredths
1	3	.	5	
→	→	.	→	
	1	.	3	5

The 1 moves one place to the right into the Units column. The 3 moves one place to the right over the decimal point into the Tenths column. The 5 moves one place into the Hundredths column. The number becomes 1.35.

$$13.5 \div 10 = 1.35$$

$2.76 \div 10 =$  We write 2.76 in the place value table. Each digit moves one place to the right. .276 is the same as 0.276.

$$2.76 \div 10 = 0.276$$

**Questions**

1.  $32.4 \div 10$
3.  $32.91 \div 10$
5.  $236.1 \div 10$
7.  $61.8 \div 10$
9.  $12.6 \div 10$
11.  $60.4 \div 10$

2.  $77.8 \div 10$
4.  $62.58 \div 10$
6.  $189.3 \div 10$
8.  $34.7 \div 10$
10.  $19.7 \div 10$
12.  $90.5 \div 10$

13.  $5.7 \div 10$

15.  $4.68 \div 10$

17.  $414.6 \div 100$

19.  $8172.1 \div 100$

21.  $362.4 \div 100$

23.  $152.9 \div 100$

25.  $26.1 \div 100$

27.  $61.8 \div 100$

29.  $1542.6 \div 1000$

31.  $6012.4 \div 1000$

33.  $52.7 \div 1000$

14.  $3.1 \div 10$

16.  $6.91 \div 10$

18.  $601.2 \div 100$

20.  $2410.8 \div 100$

22.  $787.8 \div 100$

24.  $62.5 \div 100$

26.  $189.3 \div 100$

28.  $304.7 \div 100$

30.  $1879.7 \div 1000$

32.  $920.5 \div 1000$

34.  $3.1 \div 1000$

Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)

## Chapter 23

# Rounding to the Nearest 10, 100 and 1000

### Grade 2

To round a number to the nearest ten, count up in tens. Which ten is your number closer to? For example, if your number is 23 then it is between 20 and 30. 23 is closer to 20. If your number is in the middle then round up, for example if your number is 25 then round it to 30.

#### Example 23.1

Round these numbers to the nearest ten.

23

27

15



(a) 23 to the nearest ten is 20.

(b) 27 to the nearest ten is 30.



15 is in the middle of 10 and 20, so round up to 20.

15 to the nearest ten is 20.

#### Questions A

Round these numbers to the nearest ten.

- |        |        |
|--------|--------|
| 1. 18  | 2. 82  |
| 3. 67  | 4. 36  |
| 5. 57  | 6. 79  |
| 7. 45  | 8. 5   |
| 9. 78  | 10. 22 |
| 11. 23 | 12. 53 |

#### Hints / Notes

Count up in tens, which ten is your number closer to? If your number is in the middle, then choose the larger ten. What about single digit numbers like 5? 5 is between 0 and 10.

To round a number to the nearest hundred, count up in hundreds, which hundred is your number closer to? For example, if your number is 210 then it is between 200 and 300. 210 is closer to 200. If the number is in the middle then round up, for example if your number is 250 then round it to 300.

**Example 23.2**

210

260



(a) 210 to the nearest 100 is 200.

(b) 260 to the nearest 100 is 300.

750



750 is in the middle of 700 and 800, so round up to 800.

750 to the nearest hundred is 800.

**Questions B**

Round these numbers to the nearest hundred.

- |         |         |
|---------|---------|
| 1. 160  | 2. 390  |
| 3. 570  | 4. 550  |
| 5. 380  | 6. 320  |
| 7. 140  | 8. 940  |
| 9. 450  | 10. 980 |
| 11. 230 | 12. 270 |

**Hints / Notes**

Count up in hundreds, which hundred is your number closer to? If your number is in the middle, then choose the largest hundred.

To round a number to the nearest thousand, count up in thousands, which thousand is your number closer to? For example, if your number is 5800 then it is between 5000 and 6000. 5800 is closer to 6000. If the number is in the middle then round up, for example if your number is 5500 then round it to 6000.

**Example 23.3**

Round these numbers to the nearest thousand.

6400

6800



(a) 6400 to the nearest thousand is 6000.

(b) 6800 to the nearest thousand is 7000.

3500



3500 is in the middle of 3000 and 4000, so round up to 4000.

3500 to the nearest thousand is 4000.

### Questions C

Round these numbers to the nearest thousand.

- |          |          |
|----------|----------|
| 1. 3400  | 2. 2800  |
| 3. 6700  | 4. 5200  |
| 5. 8100  | 6. 5500  |
| 7. 7500  | 8. 9300  |
| 9. 1200  | 10. 6400 |
| 11. 3700 | 12. 9500 |

### Hints / Notes

Count up in thousands, which thousand is your number closer to? If your number is in the middle, then choose the largest thousand.

## Chapter 24

# Addition and Subtraction

To find the sum of a group of numbers, we use mental methods and written methods.

## 24.1 Adding Whole Numbers Mental Method

Grade 2

Whole numbers are added by splitting each number into its hundreds, tens, units and so on. The hundreds, tens and units can be added separately and then the results can be added. This is called partitioning.

### Example 24.1

$$32 + 57 =$$

$$64 + 28 =$$

$$283 + 468 =$$

$$30 + 50 = 80$$

$$2 + 7 = 9$$

$$80 + 9 = 89$$

$$32 + 57 = 89$$

$$60 + 20 = 80$$

$$4 + 8 = 12$$

$$80 + 12 = 92$$

$$64 + 28 = 92$$

$$200 + 400 = 600$$

$$80 + 60 = 140$$

$$3 + 8 = 11$$

$$600 + 140 + 11 = 751$$

$$283 + 468 = 751$$

### Questions

1.  $12 + 56$

3.  $82 + 68$

5.  $36 + 55$

7.  $487 + 246$

9.  $381 + 749$

2.  $71 + 27$

4.  $43 + 97$

6.  $74 + 29$

8.  $286 + 124$

## 24.2 Adding Whole Numbers Written Method

### Grade 2

We can add numbers by writing them in columns. We think of the place value table and start by adding the digits on the right. If the sum is greater than nine, we add one to the column on the left.

#### Example 24.2

$$316 + 32 =$$

	Hun-	Tens	Units
	dreds	dreds	dreds
	3	1	6
+		3	2
	3	4	8

$$758 + 327 =$$

	Hun-	Tens	Units
	dreds	dreds	dreds
	7	5	8
+	3	2	7
	10	8	5
		1	

$$452 + 377 =$$

	Hun-	Tens	Units
	dreds	dreds	dreds
	4	5	2
+	3	7	7
	8	2	9
		1	

Add up the Units:

$$6 + 2 = 8$$

Put 8 in the Units column.

Add up the Tens:

$$1 + 3 = 4$$

Put 4 in the Tens column.

Put 3 in the Hundreds column.

$$316 + 32 = 348$$

Add up the Units:

$$8 + 7 = 15.$$

Put 1 in the Tens column beneath the answer box. Put 5 in the Units column.

Add up the Tens:

$$5 + 2 = 7 \text{ plus, the } 1 \text{ below } = 8$$

Put 8 in the Tens column.

Add up the Hundreds:

$$7 + 3 = 10$$

Put 10 in the Hundreds column.

$$758 + 327 = 1085$$

Add up the Units:

$$2 + 7 = 9$$

Put 9 in the Units column.

Add up the Tens:

$$5 + 7 = 12$$

Put 1 in the Hundreds column beneath the answer box. Put 2 in the Tens column.

Add up the Hundreds:

$$4 + 3 = 7 \text{ plus, the } 1 \text{ below } = 8$$

Put 8 in the Hundreds column.

$$452 + 377 = 829$$

## Questions

The Hundreds, Tens and Units have been taken off these questions, but if it helps, add them yourself.

1.  $432 + 16$
3.  $524 + 618$
5.  $386 + 235$
7.  $507 + 818$
9.  $734 + 907$

2.  $348 + 25$
4.  $653 + 175$
6.  $752 + 190$
8.  $912 + 68$

Subtraction means to find the difference between numbers. We use mental methods and written methods.

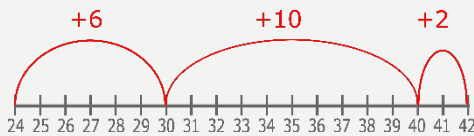
## 24.3 Subtracting Whole Numbers Mental Method

### Grade 2

We can subtract by using a number line. We start at the small number and add simple amounts until we reach the large number. The sum of the added numbers is the answer.

### Example 24.3

$$42 - 24 =$$



Start at 24, add 6 to reach 30, add 10 to reach 40, add 2 to reach 42.

$$+ 6 = 30$$

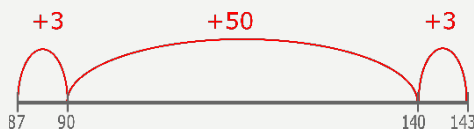
$$+ 10 = 40$$

$$+ 2 = 42$$

$$6 + 10 + 2 = 18$$

$$42 - 24 = 18$$

$$143 - 87 =$$



Start at 87, add 3 to reach 90, add 50 to reach 140, add 3 to reach 143.

$$+ 3 = 90$$

$$+ 50 = 140$$

$$+ 3 = 143$$

$$3 + 50 + 3 = 56$$

$$143 - 87 = 56$$

## Questions A

Draw a number line to help with these subtractions.

1.  $81 - 54$

2.  $107 - 89$

$$\begin{aligned} 3. & 56 - 28 \\ 5. & 86 - 37 \\ 7. & 145 - 79 \\ 9. & 275 - 88 \end{aligned}$$

$$\begin{aligned} 4. & 73 - 45 \\ 6. & 123 - 68 \\ 8. & 231 - 187 \\ 10. & 373 - 245 \end{aligned}$$

We can subtract by writing notes. We start at the smaller number and add up in steps until the larger number is reached. The answer is the sum of the numbers that have been added.

#### Example 24.4

$$82 - 51 =$$

Start at 51:

$$+ 9 = 60$$

$$+ 20 = 80$$

$$+ 2 = 82$$

$$+ 9 + 20 + 2 = 31$$

$$82 - 51 = 31$$

$$74 - 37 =$$

Start at 37:

$$+ 3 = 40$$

$$+ 30 = 70$$

$$+ 4 = 74$$

$$+ 3 + 30 + 4 = 37$$

$$74 - 37 = 37$$

$$103 - 45 =$$

Start at 45:

$$+ 5 = 50$$

$$+ 50 = 100$$

$$+ 3 = 103$$

$$+ 5 + 50 + 3 = 58$$

$$103 - 45 = 58$$

#### Questions B

$$\begin{aligned} 1. & 67 - 24 \\ 3. & 91 - 38 \\ 5. & 125 - 77 \\ 7. & 85 - 17 \\ 9. & 112 - 43 \end{aligned}$$

$$\begin{aligned} 2. & 78 - 36 \\ 4. & 42 - 19 \\ 6. & 173 - 56 \\ 8. & 126 - 58 \end{aligned}$$

**Hints / Notes**

Mental methods are methods that you can do in your head with practice!

*Trouble with this? Jump to:* **Subtracting Whole Numbers Mental Method** (p. 162)

## 24.4 Subtracting Whole Numbers Written Method

**Grade 2**

We can subtract numbers in columns. We think of the place value table and start with the column on the right. For each column, we check the top number and the number below. If the top number is larger, we subtract. If not, then we take ten from the column on the left before subtracting.

**Example 24.5**

$$365 - 42 =$$

	Hun-	Ten-	Units
	dreds		
3	6	5	
-		4	2
	3	2	3

$$753 - 326 =$$

	Hun-	Ten-	Units
	dreds		
7	5	3	
-	3	2	6
	4	2	7

$$748 - 275 =$$

	Hun-	Ten-	Units
	dreds		
7	4	8	
-	2	7	5
	4	7	3

Check the Units column, 5 is more than 2 so subtract normally:  $5 - 2 = 3$

Put 3 in the Units column.

Check the Tens column, 6 is more than 4 so subtract normally:  $6 - 4 = 2$

Put 2 in the Tens column.

Put 3 in the Hundreds column.

$$365 - 42 = 323$$

Check the Units column, 3 is *less than* 6.

Take ten from the Tens column into the Units column. The 3 becomes 13. Reduce the number in the Tens column by one. The 5 becomes 4.

Subtract the units:  $13 - 6 = 7$ .

Put 7 in the Units column.

Check the Tens column. 4 is more than 2 so subtract normally:  $4 - 2 = 2$ .

Put 2 in the Tens column.

Check the Hundreds column, 7 is more than 3 so subtract normally:  $7 - 3 = 4$

Put 4 in the Hundreds column.

$$753 - 326 = 427$$

Check the Units column, 8 is more than 5 so subtract normally:  $8 - 5 = 3$

Put 3 in the Units column.

Check the Tens column, 4 is *less than* 7.

Take a hundred from the Hundreds column into the Tens column. The 4 becomes 14. Reduce the number in the Hundreds column by one. The 7 becomes 6.

Subtract the Tens:  $14 - 7 = 7$

Put 7 in the Tens column.

Check the Hundreds column, 6 is more than 2 so subtract normally:  $6 - 2 = 4$

Put 4 in the Hundreds column.

$$748 - 275 = 473$$

## Questions

1.  $397 - 153$

3.  $528 - 148$

5.  $386 - 173$

7.  $734 - 307$

9.  $407 - 318$

2.  $348 - 25$

4.  $653 - 175$

6.  $753 - 192$

8.  $912 - 468$

### Hints / Notes

One hundred is the same as ten tens!

Trouble with this? Jump to: **Subtracting Whole Numbers Mental Method** (p. 162)

We can add decimal numbers using mental methods or written methods.

## 24.5 Adding Decimal Numbers Mental Method

### Grade 2

To add decimal numbers mentally, we can split up the numbers. We try to make whole numbers first, then we add the remaining decimal.

### Example 24.6

$$0.3 + 0.8 =$$

Make whole numbers first:

$$0.3 + 0.7 = 1.0$$

There is 0.1 remaining:

$$1.0 + 0.1 = 1.1$$

$$0.3 + 0.8 = 1.1$$

$$0.5 + 0.9 =$$

Make whole numbers first:

$$0.5 + 0.5 = 1.0$$

There is 0.4 remaining:

$$1.0 + 0.4 = 1.4$$

Alternative method:

$$5 + 9 = 14$$

The answer is 1.4

$$0.5 + 0.9 = 1.4$$

$$6.7 + 7.9 =$$

Make whole numbers first:

$$6.7 + 0.3 = 7$$

There is 7.6 remaining:

$$7 + 7.6 = 14.6$$

$$6.7 + 7.9 = 14.6$$

$$3.5 + 0.5 = 4$$

$$4 + 5.3 = 9.3$$

$$3.5 + 5.8 =$$

Alternative method:

$$35 + 58 = 93$$

The answer is 9.3

$$3.5 + 5.8 = 9.3$$

## Questions

1.  $0.7 + 0.5$

3.  $0.9 + 0.4$

5.  $5.3 + 2.7$

7.  $0.3 + 2.7$

9.  $0.4 + 0.2$

11.  $0.6 + 0.4$

2.  $0.8 + 0.6$

4.  $3.2 + 2.1$

6.  $8.4 + 9.1$

8.  $3.9 + 0.5$

10.  $2.6 + 4.6$

12.  $8.3 + 5.9$

**Hints / Notes**

You can instead imagine that the decimal numbers are whole numbers. See the alternative methods in the examples! You can still make a little note of the numbers when adding mentally, it doesn't mean that you can't write anything down!

*Trouble with this? Jump to: **Adding Whole Numbers Mental Method** (p. 159)*

## 24.6 Adding Decimal Numbers Written Method

### Grade 2

We can use columns to add decimal numbers. When adding numbers in columns we think of the place value table. We write the numbers beneath each other, lining them up on the decimal point. We start with the column on the right.

#### Example 24.7

$$12.3 + 1.9 =$$

	Tens	Units	Tenths
	1	2	. 3
+		1	. 9
	1	4	. 2
		1	

$$6.5 + 4.98 =$$

	Units	Tenths	Hun-
			dredths
	6	. 5	
+	4	. 9	8
	11	. 4	8
		1	

Starting in the Tenths column:

$$3 + 9 = 12$$

Put 1 in the Units column beneath the answer box. Put 2 in the Tenths column.

Add up the Units:

$$2 + 1 = 3 \text{ plus the 1 below} = 4$$

Put 4 in the Units column.

Put 1 in the Tens column.

$$12.3 + 1.9 = 14.2$$

Put 8 in the Hundredths column.

Add up the Tenths:

$$5 + 9 = 14$$

Put 1 in the Units column beneath the answer box. Put 4 in the Tenths column.

Add up the Units:

$$6 + 4 = 10 \text{ plus the 1 below} = 11$$

Put 11 in the Units column.

$$6.5 + 4.98 = 11.48$$

## Questions

The Hundreds, Tens and Units have been taken off these questions, but if it helps, add them yourself. You can replace spaces in the columns with zeros.

1.  $7.3 + 0.16$
3.  $37.1 + 14.9$
5.  $38.0 + 2.7$
7.  $50.7 + 8.1$
9.  $9.3 + 17.0$

2.  $54.8 + 2.7$
4.  $9.53 + 7.8$
6.  $8.04 + 0.68$
8.  $9.12 + 6.85$

We can subtract decimal numbers using mental methods or written methods.

## 24.7 Subtracting Decimal Numbers Mental Method

Grade 2

To subtract decimal numbers mentally, we can split the numbers into whole numbers and decimals. We subtract the whole number first, then we subtract the decimal part.

### Example 24.8

$$6.8 - 2.3 =$$

Split 2.3 into 2 and 0.3:

$$6.8 - 2 = 4.8$$

$$4.8 - 0.3 = 4.5$$

$$6.8 - 2.3 = 4.5$$

$$7.3 - 5.7 =$$

Split 5.7 into 5 and 0.7:

$$7.3 - 5 = 2.3$$

$$2.3 - 0.7 = 1.6 \text{ (same as } 2.3 - 0.3 - 0.4)$$

$$7.3 - 5.7 = 1.6$$

An alternative method is to count up from the small number by adding decimals.

$$8.7 - 4.9 =$$

Count up from 4.9:

$$+ 0.1 = 5.0$$

$$+ 3 = 8.0$$

$$+ 0.7 = 8.7$$

$$0.1 + 3 + 0.7 = 3.8$$

$$8.7 - 4.9 = 3.8$$

## Questions

1.  $0.9 - 0.2$
3.  $5.8 - 1.7$
5.  $3.4 - 2.5$
7.  $8.6 - 2.1$
9.  $8.2 - 3.4$

2.  $8.4 - 0.3$
4.  $7.4 - 5.1$
6.  $7.4 - 3.8$
8.  $7.3 - 5.6$

#### Hints / Notes

If you prefer, you can imagine that the numbers are whole numbers, for example  $2.3 - 0.7$  is similar to  $23 - 7$  which is 16. The answer is 1.6! You can still make a little note of the numbers when subtracting mentally, it doesn't mean that you can't write anything down!

Trouble with this? Jump to: **Subtracting Whole Numbers Mental Method** (p. 162)

## 24.8 Subtracting Decimal Numbers Written Method

### Grade 2

We can subtract decimal numbers in columns. When subtracting in columns, we think of the place value table. We write the numbers beneath each other, lining them up on the decimal point. We start on the right.

#### Example 24.9

$$81.4 - 2.3 =$$

Tens	Units	Tenths	
8	1	4	
-	2	3	
7	9	1	

Starting on the right in the Tenths column:

$$4 - 3 = 1$$

Put 1 in the Tenths column.

Check the Units column. *1 is less than 2.*

Take ten from the Tens column into the Units column. The 1 becomes 11. Reduce the number in the Tens column by one, the 8 becomes 7.

$$\text{Subtract the units: } 11 - 2 = 9$$

Put 9 in the Units column.

Put 7 in the Tens column.

$$81.4 - 2.3 = 79.1$$

Check the Hundredths column. *6 is less than 8.* Take one tenth from the Tenths column into the Hundredths column. The 6 becomes 16. Reduce the number in the Tenths column by one, the 3 becomes 2.

$$\text{Subtract the Hundredths: } 16 - 8 = 8.$$

Put 8 in the Hundredths column.

Check the Tenths column. Subtract the Tenths:  $2 - 2 = 0$

Put 0 in the Tenths column.

Check the Units column. Subtract the Units:

$$9 - 0 = 9$$

Put 9 in the Units column.

$$9.36 - 0.28 = 9.08$$

Check the Hundredths column. *0 is less than 3.*

The tenths column has a 0, so we cannot take from the tenths. Reduce the number in the *Units* column by one, the 5 becomes 4.

Take the unit from the Units column into the Tenths column. The 0 becomes 10. Take the tenth from the Tenths column into the Hun-

$$9.36 - 0.28 =$$

Units	Tenths	Hun-	
		dredths	
9	3	6	
-	0	2	8
9	0	8	

$$25.00 - 4.23 =$$

Tens	Units	Tenths	Hun-	
			dredths	
2	5	0	0	
-	0	4	2	3
2	0	7	7	

171 hundredths column. The 0 becomes 10. Reduce the number in the Tenths column by one, the 10 becomes 9.

## Questions

You can replace any spaces in the columns with zeros.

1.  $9.34 - 4.21$

3.  $15.9 - 7.6$

5.  $38.4 - 19.3$

7.  $40.7 - 8.2$

9.  $2.00 - 1.59$

2.  $64.0 - 2.3$

4.  $8.63 - 4.56$

6.  $2.78 - 0.91$

8.  $9.00 - 6.81$

### Hints / Notes

You can replace any spaces in the columns with zeros. If the top number has more than one zero, you can instead adjust each number first. See the alternative method.

*Trouble with this? Jump to: **Subtracting Whole Numbers Mental Method** (p. 162)*

## Chapter 25

# Multiplication and Division

## 25.1 Multiplying Using the Times Tables or By Counting On

Grade 2

To multiply small numbers, we can count in our heads or use our fingers. For example, to know the answer to  $8 \times 3$  we count up in 3's.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36

$$8 \times 3 = 24$$

To multiply by larger numbers, we write down the times table, then count down the list. For example, to know the answer to  $7 \times 6$ , we write down the six times table.

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72

Counting along 7 places, the answer to  $7 \times 6$  is 42.

### Questions

$1.5 \times 5$	$2.4 \times 5$
$3.6 \times 3$	$4.3 \times 7$
$5.4 \times 9$	$6.8 \times 9$
$7.7 \times 5$	$8.8 \times 7$
$9.4 \times 8$	$10.2 \times 4$
$11.5 \times 9$	$12.6 \times 4$
$13.11 \times 4$	$14.2 \times 6$
$15.8 \times 6$	$16.6 \times 9$
$17.9 \times 8$	$18.4 \times 3$
$19.7 \times 4$	$20.6 \times 6$
$21.12 \times 3$	$22.4 \times 7$
$23.9 \times 7$	$24.9 \times 9$
$25.5 \times 6$	$26.10 \times 4$
$27.4 \times 11$	$28.7 \times 11$
$29.1 \times 6$	$30.3 \times 3$
$31.5 \times 8$	$32.8 \times 8$
$33.8 \times 3$	$34.4 \times 12$
$35.2 \times 7$	$36.2 \times 11$
$37.6 \times 7$	$38.7 \times 7$
$39.2 \times 9$	$40.12 \times 9$

### Hints / Notes

To multiply by 4, double the number twice. To multiply by 8, double the number three times! You should be able to count up in 2's, 3's, 4's, 5's and 10's in your head. If you can't, then practice every day until you can!

## 25.2 Multiplying Whole Numbers by Single Digits

### Grade 2

To multiply numbers that end in zero, we multiply by the single digit, then we multiply by the multiple of ten.

#### Example 25.1

$40 \times 5 = 200$       40 is  $4 \times 10$ . Multiply 4 by 5, then multiply the answer by 10:  $4 \times 5 = 20$ , and  $20 \times 10 = 200$ .

$50 \times 6 = 300$       50 is  $5 \times 10$ .  $5 \times 6 = 30$ , and  $30 \times 10 = 300$ .

$700 \times 6 = 4200$       700 is  $7 \times 100$ .  $7 \times 6 = 42$ , and  $42 \times 100 = 4200$ .

### Questions A

1.  $60 \times 3$

3.  $70 \times 4$

5.  $6 \times 40$

7.  $200 \times 6$

9.  $7 \times 500$

2.  $30 \times 7$

4.  $90 \times 2$

6.  $7 \times 50$

8.  $3 \times 800$

10.  $4 \times 300$

#### Hints / Notes

Multiples of ten are 10, 100, 1000 and so on. Start off with something you know — the times tables — then multiply by ten!

*Trouble with this? Jump to: **Multiplying Whole Numbers by Powers of Ten** (p. 150)*

If the numbers do not end in zero, then we split up the number. We multiply each part separately, then add the results. This is called partitioning.

#### Example 25.2

$46 \times 5 = 230$       46 is  $40 + 6$ . Multiply 40 by 5, then 6 by 5:  $40 \times 5 = 200$  and  $6 \times 5 = 30$ .

Add the answers:  $200 + 30 = 230$ .

$78 \times 6 = 468$       78 is  $70 + 8$ .  $70 \times 6 = 420$  and  $8 \times 6 = 48$ .  $420 + 48 = 468$ .

$243 \times 6 = 1458$       243 is  $200 + 40 + 3$ .  $200 \times 6 = 1200$ ,  $40 \times 6 = 240$ ,  $3 \times 6 = 18$ .  $1200 + 240 + 18 = 1458$ .

### Questions B

1.  $62 \times 3$

3.  $72 \times 4$

5.  $4 \times 67$

7.  $8 \times 56$

9.  $731 \times 4$

11.  $6 \times 572$

2.  $38 \times 5$

4.  $95 \times 2$

6.  $5 \times 78$

8.  $5 \times 94$

10.  $262 \times 5$

12.  $8 \times 249$

**Hints / Notes**

Split up the bigger number, then multiply each part.

*Trouble with this? Jump to: **Multiplying Using the Times Tables or By Counting On** (p. 173)*

## 25.3 Multiplying Whole Numbers by Two Digits Column Method

**Grade 2**

To multiply by two-digit numbers we can write the numbers in a column, beneath each other. We multiply the top number by the units of the bottom number, then by the tens and so on. We then add the results.

**Example 25.3**

$$36 \times 25 =$$

$$\begin{array}{r} 36 \\ \times 25 \\ \hline 180 \\ 720 \\ \hline 900 \end{array}$$

$$425 \times 32 =$$

$$\begin{array}{r} 425 \\ \times 32 \\ \hline 850 \\ 12750 \\ \hline 13600 \end{array}$$

Multiply the top number by 5:

$$36 \times 5 = 180$$

Write the result, 180, in the first row of the answer box.

Multiply the top number by 20:

$$36 \times 20 = 720$$

Write the result, 720, in the second row of the answer box.

Add up the results in the answer box:

$$180 + 720 = 900$$

$$36 \times 25 = 900$$

Multiply the top number by 2:

$$425 \times 2 = 850$$

Write the answer, 850, in the first row of the answer box.

Multiply the top number by the 30:

$$425 \times 30 = 12750$$

Write the answer, 12750, in the second row of the answer box.

Add up the results in the answer box:

$$850 + 12750 = 13600$$

$$425 \times 32 = 13600$$

## 25.4 Multiplying Whole Numbers by Two Digits Table Method

### Grade 2

To multiply by two-digit numbers we can use a table. We split up the numbers into Units, Tens and so on. We multiply each part separately, writing the results in the table. We then add up the results.

#### Example 25.4

$$36 \times 25 =$$

$$\begin{array}{r} \times 30 \quad 6 \\ 20 \quad 600 \quad 120 \\ 5 \quad 150 \quad 30 \end{array}$$

$$425 \times 83 =$$

$$\begin{array}{r} \times 400 \quad 20 \quad 5 \\ 80 \quad 32000 \quad 1600 \quad 400 \\ 3 \quad 1200 \quad 60 \quad 15 \end{array}$$

30 is split into 30 and 6.

25 is split into 20 and 5.

$$30 \times 20 = 600$$

$$6 \times 20 = 120$$

$$30 \times 5 = 150$$

$$6 \times 5 = 30$$

Add the results:  $600 + 120 + 150 + 30 =$

$$900$$

$$36 \times 25 = 900$$

425 is split into 400, 20 and 5.

83 is split into 80 and 3.

$$400 \times 80 = 32000$$

$$20 \times 80 = 1600$$

$$5 \times 80 = 400$$

$$400 \times 3 = 1200$$

$$20 \times 3 = 60$$

$$5 \times 3 = 15$$

Add the results:  $32000 + 1600 + 400 +$

$$1200 + 60 + 15 = 35275$$

$$425 \times 83 = 35275$$

### Questions — try both methods

1.  $46 \times 32$

3.  $82 \times 45$

5.  $48 \times 39$

7.  $238 \times 56$

9.  $452 \times 61$

2.  $37 \times 51$

4.  $91 \times 37$

6.  $56 \times 28$

8.  $423 \times 27$

10.  $528 \times 43$

#### Hints / Notes

You can multiply the single digits, then add one zero for each zero in the numbers!

$400 \times 80 = 4 \times 8$  plus a zero for each zero in the numbers = 32000! You can also add the

*Trouble with this? Jump to: **Multiplying Using the Times Tables or By Counting On** (p. 173); **Adding Whole Numbers Written Method** (p. 160)*

results by writing them beneath each other  
in a column.

## 25.5 Dividing Using the Times Tables or By Counting On

### Grade 2

To divide small numbers, we can count in our heads and use our fingers. For example, to find the answer to  $27 \div 3$ , we start at 3 and count on in 3's until we get to 27.

3, 6, 9, 12, 15, 18, 21, 24, 27

The answer is 9.

To divide larger numbers, we write out the times table. For example, to find the answer to  $42 \div 7$ , we write out the 7 times table. We start at 7, then we add 7 each time:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84

We count along until we get to 42, it is the 6<sup>th</sup> number in the list, the answer is 6.

### Questions

- |                 |                  |
|-----------------|------------------|
| 1. $5 \div 5$   | 2. $45 \div 5$   |
| 3. $18 \div 9$  | 4. $42 \div 7$   |
| 5. $63 \div 9$  | 6. $40 \div 4$   |
| 7. $55 \div 5$  | 8. $28 \div 7$   |
| 9. $12 \div 2$  | 10. $20 \div 4$  |
| 11. $27 \div 9$ | 12. $16 \div 8$  |
| 13. $16 \div 4$ | 14. $24 \div 6$  |
| 15. $12 \div 2$ | 16. $56 \div 7$  |
| 17. $24 \div 8$ | 18. $40 \div 5$  |
| 19. $2 \div 2$  | 20. $60 \div 6$  |
| 21. $45 \div 9$ | 22. $14 \div 7$  |
| 23. $9 \div 3$  | 24. $63 \div 9$  |
| 25. $81 \div 9$ | 26. $44 \div 4$  |
| 27. $40 \div 5$ | 28. $77 \div 11$ |
| 29. $18 \div 6$ | 30. $36 \div 3$  |
| 31. $25 \div 5$ | 32. $32 \div 4$  |
| 33. $8 \div 4$  | 34. $40 \div 5$  |
| 35. $21 \div 7$ | 36. $21 \div 3$  |
| 37. $6 \div 2$  | 38. $32 \div 8$  |
| 39. $20 \div 5$ | 40. $72 \div 6$  |
| 41. $32 \div 4$ | 42. $36 \div 6$  |
| 43. $24 \div 2$ | 44. $32 \div 8$  |
| 45. $24 \div 3$ | 46. $72 \div 6$  |

## 25.6 Dividing Whole Numbers by Single Digits

### Grade 2

Dividing means sharing into equal amounts. Sometimes when we share, there is an amount left over, this is called the remainder.

If we divide twelve buns between three people, then they each have four buns.  $12 \div 3 = 4$ .

If instead, we divide twelve buns between five people, each person has two buns and there are two left over.  $12 \div 5$  is 2 remainder 2.

We write 12 divided by 3, using the bus stop method as  $3 \overline{)12}$ . The bus stop method is also called the short division method.

**Example 25.5**

$$484 \div 4 =$$

Write as a bus stop:

$$4 \overline{) 484}$$

$$378 \div 3 =$$

Write as a bus stop:

$$3 \overline{) 378}$$

$$925 \div 4 =$$

Write as a bus stop:

$$4 \overline{) 925}$$

$$378 \div 5 =$$

Write as a bus stop:

$$5 \overline{) 378}$$

4 goes into 4 once. Place 1 above the first 4.

4 goes into 8 twice. Place 2 above the 8.

4 goes into 4 once. Place 1 above the 4.

$$\begin{array}{r} 121 \\ 4 \overline{) 484} \end{array}$$

$$484 \div 4 = 121$$

3 goes into 3 once. Place 1 above the 3.

3 goes into 7 twice with remainder 1.

Place 2 above the 7 and place 1 in front of the next digit, the 8 becomes 18.

3 goes into 18 six times. Place 6 above the 18.

$$\begin{array}{r} 126 \\ 3 \overline{) 378} \end{array}$$

$$378 \div 3 = 126$$

4 goes into 9 twice with remainder 1. Place 2 above the 9 and place 1 in front of the next digit. The 2 becomes 12.

4 goes into 12 three times. Place 3 above the 12.

4 goes into 5 once with remainder 1. Place 1 above the 4, then write r1 (remainder 1).

$$\begin{array}{r} 231 \text{ r}1 \\ 4 \overline{) 925} \end{array}$$

$$925 \div 4 = 231 \text{ r}1$$

5 goes into 3 zero times with remainder 3. Place 0 above the 3. Place 3 in front of the next digit. The 7 becomes 37.

5 goes into 37 seven times with remainder 2. Place 7 above the 37. Place 2 in front of the next digit. The 8 becomes 28.

5 goes into 28 five times with remainder 3. Place 5 above the 28, then write r3 (remainder 3).

$$\begin{array}{r} 075 \text{ r}3 \\ 5 \overline{) 378} \end{array}$$

**Questions**

1. 
$$\begin{array}{r} \overline{2)68} \end{array}$$

3. 
$$\begin{array}{r} \overline{3)72} \end{array}$$

5. 
$$\begin{array}{r} \overline{4)316} \end{array}$$

7. 
$$\begin{array}{r} \overline{6)865} \end{array}$$

9. 
$$\begin{array}{r} \overline{7)919} \end{array}$$

2. 
$$\begin{array}{r} \overline{3)63} \end{array}$$

4. 
$$\begin{array}{r} \overline{4)724} \end{array}$$

6. 
$$\begin{array}{r} \overline{5)280} \end{array}$$

8. 
$$\begin{array}{r} \overline{8)906} \end{array}$$

10. 
$$\begin{array}{r} \overline{9)769} \end{array}$$

**Hints / Notes**

Remember to carry any remainder to the front of the next number.

*Trouble with this? Jump to: **Dividing Using the Times Tables or By Counting On** (p. 178)*

**25.7 Dividing Whole Numbers by Two Digits Chunking Method****Grade 2**

Division can be thought of as repeated subtraction. If we want to know how many times a number goes into something we can keep taking the number away. The amount of times we do this is the answer.

For example, if we want to know how many times 5 goes into 21, we can keep subtracting 5:

$21 - 5 = 16$	Once
$16 - 5 = 11$	Twice
$11 - 5 = 6$	Three times
$6 - 5 = 1$	Four times

Therefore 5 goes into 21, 4 times remainder 1.

This method is useful for dividing by two-digit numbers. Amounts (chunks) of the number are taken away until the number is too small. The amounts (chunks) are added up to find the answer. This is called chunking.

If the number is large, we may subtract ten lots or twenty lots of the number all at once.

**Example 25.6**

$$252 \div 14 =$$

Draw a bus stop:

$$\begin{array}{r} 14 \overline{) 252} \\ - 140 \quad -10 \times 14 \\ \hline 112 \\ - 70 \quad -5 \times 14 \\ \hline 42 \\ - 42 \quad -3 \times 14 \\ \hline 0 \end{array}$$

$$546 \div 17 =$$

Draw as a bus stop:

$$\begin{array}{r} 17 \overline{) 546} \\ - 340 \quad -20 \times 17 \\ \hline 206 \\ - 170 \quad -10 \times 17 \\ \hline 036 \\ - 034 \quad -2 \times 17 \\ \hline 2 \end{array}$$

Subtract 10 lots of 14,  $10 \times 14 = 140$ .

There is 112 remaining.

Subtract 5 lots of 14,  $5 \times 14 = 70$ .

There is 42 remaining.

Subtract 3 lots of 14,  $3 \times 14 = 42$ .

We are left with zero.

Add up the amounts of 14:

$$10 + 5 + 3 = 18$$

$$252 \div 14 = 18$$

Subtract 20 lots of 17,  $20 \times 17 = 340$ .

There is 206 remaining.

Subtract 10 lots of 17,  $10 \times 17 = 170$ .

There is 36 remaining.

Subtract 2 lots of 17,  $2 \times 17 = 34$ .

There is 2 remaining.

Add up the amounts of 17:

$$20 + 10 + 2 = 32$$

$$546 \div 17 = 32 \text{ remainder } 2.$$

## Questions

1.  $13 \overline{) 156}$

3.  $16 \overline{) 688}$

5.  $13 \overline{) 365}$

7.  $15 \overline{) 540}$

2.  $14 \overline{) 630}$

4.  $17 \overline{) 425}$

6.  $14 \overline{) 380}$

8.  $18 \overline{) 936}$

### Hints / Notes

Taking away ten lots of a number all at once saves time.

Trouble with this? Jump to: **Dividing Whole Numbers by Single Digits** (p. 178)

## 25.8 Multiplying Decimal Numbers with One Digit

### Grade 2

To multiply decimal numbers with a single non-zero digit, we multiply the single digit, then we divide the answer by a multiple of ten.

#### Example 25.7

$6 \times 0.3 = 1.8$       0.3 is 3 divided by 10.  $6 \times 3 = 18$ , then  $18 \div 10 = 1.8$ .  
 $0.7 \times 4 = 2.8$       0.7 is 7 divided by 10.  $7 \times 4 = 28$ , then  $28 \div 10 = 2.8$ .  
 $0.08 \times 5 = 0.4$       0.08 is 8 divided by 100.  $8 \times 5 = 40$ , then  $40 \div 100 = 0.4$ .

### Questions A

- |                    |                    |
|--------------------|--------------------|
| 1. $3 \times 0.5$  | 2. $0.7 \times 7$  |
| 3. $9 \times 0.7$  | 4. $0.2 \times 6$  |
| 5. $8 \times 0.4$  | 6. $4 \times 0.3$  |
| 7. $0.02 \times 4$ | 8. $6 \times 0.9$  |
| 9. $0.04 \times 5$ | 10. $3 \times 0.8$ |

*Trouble with this? Jump to: **Multiplying Whole Numbers by Single Digits** (p. 174); **Dividing Whole Numbers by Powers of Ten** (p. 151)*

If both numbers are decimal numbers, then we may have to divide by ten more than once.

#### Example 25.8

$0.8 \times 0.6 = 0.48$       0.8 is 8 divided by 10 and 0.6 is 6 divided by 10.  $8 \times 6 = 48$ . Divide the answer by 100:  $48 \div 100 = 0.48$ .  
 $0.7 \times 1.2 = 0.84$       0.7 is 7 divided by 10 and 1.2 is 12 divided by 10.  $7 \times 12 = 84$ . Divide the answer by 100:  $84 \div 100 = 0.84$ .

### Questions B

- |                     |                      |
|---------------------|----------------------|
| 1. $0.3 \times 0.5$ | 2. $0.4 \times 0.7$  |
| 3. $0.9 \times 0.6$ | 4. $0.5 \times 0.8$  |
| 5. $0.2 \times 0.9$ | 6. $0.8 \times 0.8$  |
| 7. $0.8 \times 1.2$ | 8. $1.1 \times 0.3$  |
| 9. $0.5 \times 0.9$ | 10. $1.2 \times 0.6$ |

#### Hints / Notes

Another method is to use patterns:  $0.3 \times 0.5$  is similar to  $3 \times 5$ . We can write:  $3 \times 5 = 15$ ,  $0.3 \times 5 = 1.5$ ,  $0.3 \times 0.5 = 0.15$ , dividing the answer by ten each time!

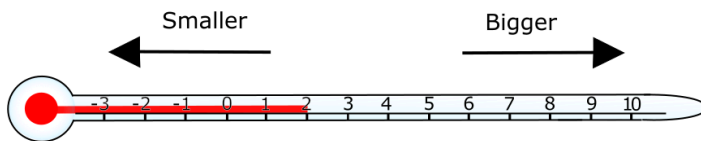
*Trouble with this? Jump to: **Multiplying Whole Numbers by Single Digits** (p. 174); **Dividing Whole Numbers by Powers of Ten** (p. 151)*

## Chapter 26

# Negative Numbers on a Thermometer

### Grade 2

The thermometer below shows that as the temperature gets colder, the numbers become smaller. When the temperature is less than zero, the numbers become negative.



As the temperature increases, we move to the right. If the temperature decreases, we move to the left.

### Example 26.1

The temperature in a barn during the night was  $-2^{\circ}\text{C}$ . In the morning, the temperature increased by  $5^{\circ}\text{C}$ .

Start at  $-2$  on the thermometer, then jump five spaces to the right.

The temperature in the morning was  $3^{\circ}\text{C}$ .

What was the temperature in the morning?

### Example 26.2

The temperature in a cool box was  $2^{\circ}\text{C}$ . Frozen peas were added to the cool box. The temperature then decreased by  $3^{\circ}\text{C}$ .

Start at  $2$  on the thermometer, then jump three spaces to the left.

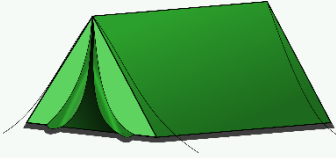
The temperature after adding the peas was  $-1^{\circ}\text{C}$ .

What was the temperature in the cool box after the frozen peas were added?

### Questions A

1. The temperature in Edinburgh city centre was  $-3^{\circ}\text{C}$  on Monday. On Tuesday, the temperature increased by  $5^{\circ}\text{C}$ . What was the temperature on Tuesday?

2. The temperature in Callum's fridge was  $4^{\circ}\text{C}$ . Callum put some frozen yogurt in the fridge. The temperature went down by  $6^{\circ}\text{C}$ . What was the new temperature?



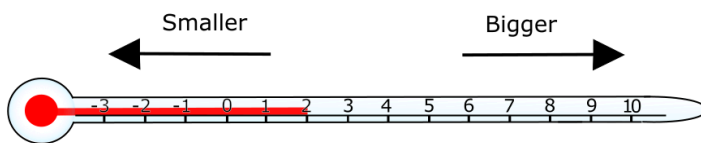
3. The temperature in Jessie's tent at night was  $-1^{\circ}\text{C}$ . In the morning the temperature increased by  $4^{\circ}\text{C}$ . What was the temperature in the morning?

4. The temperature of Mia's ice block was  $-3^{\circ}\text{C}$ . The ice block was left in Mia's bag and its temperature went up by  $3^{\circ}\text{C}$ . What was the temperature of the ice block after it had been in Mia's bag?

5. The temperature in Patrick's garden at lunchtime was  $7^{\circ}\text{C}$ . At dusk, the temperature had decreased by  $8^{\circ}\text{C}$ . What was the temperature at dusk?

6. The temperature on the wing of an aircraft was  $4^{\circ}\text{C}$ . The temperature went down by  $7^{\circ}\text{C}$  as the aircraft flew higher. What was the temperature after the aircraft flew higher?

Sometimes we are given two temperatures and we are asked to find the difference between them. We can do this by using a thermometer, or a number line.



### Example 26.3

Rohan's garden was  $-1^{\circ}\text{C}$ . Five hours later, the temperature had increased to  $7^{\circ}\text{C}$ .

What is the difference in the temperature?

We start at  $-1$  on the thermometer and jump along the numbers to  $7$ .

We count the number of jumps.

We jumped 8 times.

The difference is  $8^{\circ}\text{C}$

### Questions B

Find the difference in temperature for the following questions.

1. The temperature in Ireland on a winter's morning was  $-2^{\circ}\text{C}$ . In the afternoon, the temperature was  $4^{\circ}\text{C}$ .

2. The temperature at the park on Wednesday was  $-1^{\circ}\text{C}$ . On Thursday it was  $5^{\circ}\text{C}$ .

3. The temperature at the ice rink was  $1^{\circ}\text{C}$ . After a new layer of ice was added, the temperature was  $-4^{\circ}\text{C}$ .

4. The temperature in Daniel's car in winter was  $5^{\circ}\text{C}$ . After the door was left open, the temperature decreased to  $-3^{\circ}\text{C}$ .

5. The temperature on the deck of a ship was  $-3^{\circ}\text{C}$ . After the sun came out, the temperature



was  $8^{\circ}\text{C}$ .

6. The temperature around the pond in the garden was  $7^{\circ}\text{C}$ . A few hours later, the temperature had dropped to  $-1^{\circ}\text{C}$ .

## Chapter 27

# Factor Spiders

### Grade 2

Factors are numbers that divide into other numbers exactly, with no remainder. We know the first two factors of a number, they are 1 and the number itself. To find more factors we try all the numbers from 2 to the number, increasing by 1 each time.

We start at 2, we count up in 2's on our fingers. If we reach the number we stop, 2 is a factor. If we count past the number without reaching it, then 2 is not a factor. Next, we try 3 and so on.

We can write the factors by using a factor spider. The number is placed in the middle of the spider. Factors of the number are written at the end of the spider's legs. When we have found some factors, we can think of what numbers to multiply by to make our number. This helps us to find all of the factors.

### Example 27.1

Find the factors of 12.

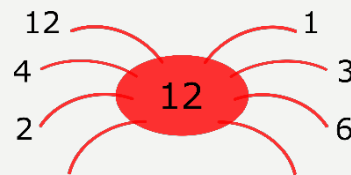
1 and 12 are factors.

2 is a factor.

$2 \times 6 = 12$  therefore 6 is a factor.

3 is a factor.

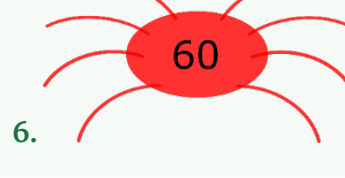
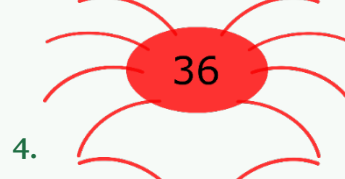
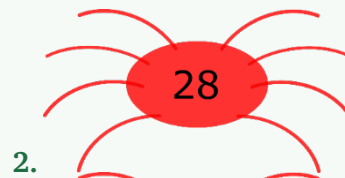
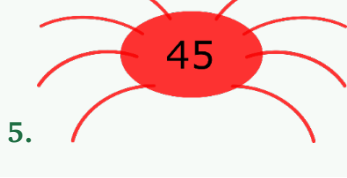
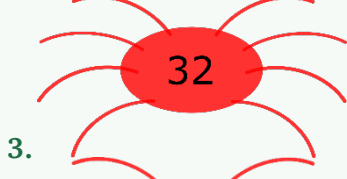
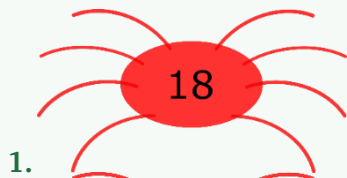
$3 \times 4 = 12$  therefore 4 is a factor.



The factors of 12 are: 1, 2, 3, 4, 6 and 12.

### Questions

Find the factors for these numbers.



**Hints / Notes**

You can count up in 2's, 3's and so on, on your fingers.

## Chapter 28

# Fractions and Decimal Numbers

## 28.1 Fraction Names

Grade 2

The top of a fraction is called the numerator. The bottom of a fraction is called the denominator. To find the fraction of an object that is shaded, we count the shaded parts, this is the numerator. We then count the total number of parts, this is the denominator.

### Example 28.1

Write down the fraction that is shaded using words and fractions.



1 shaded part. 2 parts in total. One Half.  $\frac{1}{2}$



1 shaded part. 4 parts in total. One Quarter.  $\frac{1}{4}$



Three Quarters.  $\frac{3}{4}$



One Fifth.  $\frac{1}{5}$



Three Tenths.  $\frac{3}{10}$



One Sixth.  $\frac{1}{6}$



Five Eighths.  $\frac{5}{8}$

### Questions A

Write down the fraction that is shaded using words and fractions.



8.



**Hints / Notes**

We don't say one fourth, we say one quarter instead! If there are 5 parts, they are fifths. If there are 6 parts they are sixths and so on!

The lines below have been divided into equal parts. The arrow points to a fraction along the line.

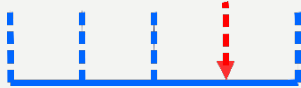
**Example 28.2**

How far along the line is the arrow?

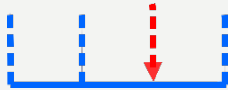


The line has 6 parts. The arrow is 1 part along.

The arrow is  $\frac{1}{6}$  of the way along the line.



The line has 4 parts. The arrow is 3 parts along. The arrow is  $\frac{3}{4}$  of the way along the line.



The arrow is  $\frac{2}{3}$  of the way along the line.

**Questions B**

How far along the line is the arrow?

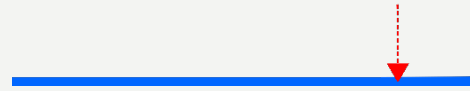


We can imagine splitting a line into separate parts. We can then draw the arrow in the correct place.

**Example 28.3**

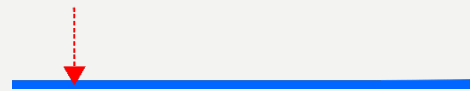
Place an arrow  $\frac{5}{6}$  of the way along this line.

Imagine splitting the line into 6 parts. Jump along 5 places then draw the arrow.



Place an arrow  $\frac{3}{10}$  of the way along this line.

Imagine splitting the line into 10 parts. Jump along 3 places then draw the arrow.



### Questions C

Place an arrow in the correct place along the line, given by the fraction.

1.  $\frac{1}{4}$



2.  $\frac{2}{5}$



3.  $\frac{7}{10}$



4.  $\frac{3}{8}$



5.  $\frac{5}{6}$



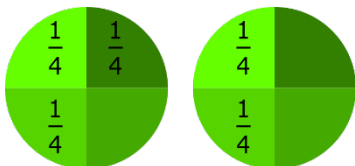
6.  $\frac{2}{3}$



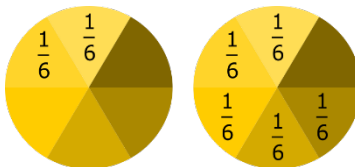
## 28.2 Ordering Fractions with the Same Denominator

Grade 3

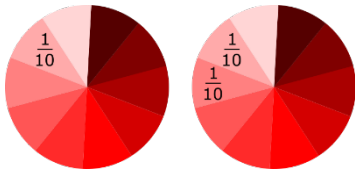
Fractions can be placed in order of size, smallest to largest. We need to know which fraction is the largest, then we can order them.



Three quarters are bigger than two quarters, the order is:  $\frac{2}{4}$   $\frac{3}{4}$



Five sixths are bigger than two sixths, the order is:  $\frac{2}{6}$   $\frac{5}{6}$



Two tenths are bigger than one tenth, the order is:  $\frac{1}{10}$   $\frac{2}{10}$

**Example 28.4**

Put these Fractions in order of size, smallest to largest.

$\frac{7}{10}$ ,  $\frac{3}{10}$ ,  $\frac{5}{10}$ ,  $\frac{1}{10}$

One tenth is the smallest, then three tenths, then five tenths and finally seven tenths. The order is:  $\frac{1}{10}$   $\frac{3}{10}$   $\frac{5}{10}$   $\frac{7}{10}$

**Questions**

Put these fractions in order of size, smallest to largest.

1.  $\frac{4}{10}$ ,  $\frac{3}{10}$ ,  $\frac{1}{10}$ ,  $\frac{2}{10}$
2.  $\frac{1}{6}$ ,  $\frac{3}{6}$ ,  $\frac{5}{6}$ ,  $\frac{4}{6}$
3.  $\frac{3}{4}$ ,  $\frac{1}{4}$ ,  $\frac{2}{4}$ ,  $\frac{4}{4}$
4.  $\frac{3}{8}$ ,  $\frac{5}{8}$ ,  $\frac{1}{8}$ ,  $\frac{7}{8}$
5.  $\frac{1}{9}$ ,  $\frac{7}{9}$ ,  $\frac{5}{9}$ ,  $\frac{2}{9}$

**Hints / Notes**

If you are not sure try drawing a circle split into the different fractions to see which one is smallest!

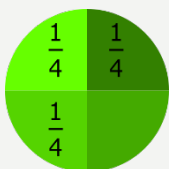
**28.3 Adding Fractions with the Same Denominator**

Grade 3

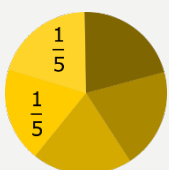
Fractions that have the same denominator can be added. We add the numerators (top) then we write the answer over the original denominator (bottom).

**Example 28.5**

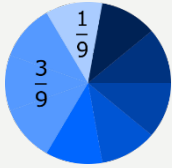
Write down the fraction sums shown in the diagrams.



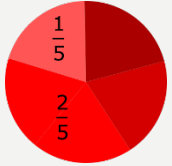
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$  Add the numerators:  $1 + 1 + 1 =$   
3. Write over the denominator:  $= \frac{3}{4}$



$\frac{1}{5} + \frac{1}{5} =$  Add the numerators:  $1 + 1 = 2$ . Write over the denominator:  $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$



$\frac{1}{9} + \frac{3}{9} =$  Add the numerators:  $1 + 3 = 4$ . Write over the denominator:  $\frac{1}{9} + \frac{3}{9} = \frac{4}{9}$



$\frac{2}{5} + \frac{1}{5} =$  Add the numerators:  $2 + 1 = 3$ . Write over the denominator:  $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

## Questions

Write down the fraction sums shown in these diagrams.



1.



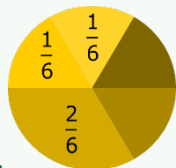
2.



3.



4.



5.



6.

## 28.4 What is a Decimal?

### Grade 2

A decimal is part of a whole number. Imagine a whole bar split into 10 equal parts. One of the parts is one tenth or 0.1. Two of the parts are 2 tenths or 0.2.

**Example 28.6**

Count the number of tenths. Write the shaded part as a decimal.



The shaded area is one tenth.  $\frac{1}{10}$  as a decimal is 0.1.



The shaded area is three tenths.  $\frac{3}{10}$  as a decimal is 0.3.



The shaded area is twelve tenths, (one whole plus two tenths).  $\frac{12}{10}$  as a decimal is 1.2.



**Questions**

Count the number of tenths. Write the shaded part as a decimal.

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.

*Trouble with this? Jump to: **Fraction Names** (p. 189)*

## Chapter 29

# Greater Than and Less Than Notation

### Grade 1

To say that something is greater than we use the symbol  $>$ . To say that something is less than, we use the symbol  $<$ . To say something is equal to, we use the symbol  $=$ .

#### Key point

Greater than  $>$

Less than  $<$

Equal to  $=$

**Example 29.1** — Write the correct sign between these expressions.

8  6      Eight is greater than six.  $8 > 6$

7  12      Seven is less than twelve.  $7 < 12$

### Questions A

Write the correct sign between these expressions.

1.  $5 \square 6$

3.  $11 \square 2$

5.  $9 \square 9$

7.  $4 \square 7$

9.  $33 \square 5$

11.  $93 \square 9$

2.  $6 \square 2$

4.  $15 \square 15$

6.  $9 \square 10$

8.  $7 \square 1$

10.  $5 \square 12$

12.  $0 \square 2$

#### Hints / Notes

For example,  $8 > 6$  (8 is greater than 6).

Imagine 8 is pointing to 6 and saying, 'you're small!'

*Trouble with this? Jump to: **Fraction Names** (p. 189)*

Sometimes we work out the calculation on each side, before adding the correct symbol.

**Example 29.2** — Write the correct sign between these expressions.

$(2 + 7) - 5 \square (7 - 6) + 2$

Work out the left-hand side:

$2 + 7 = 9$

$9 - 5 = 4$

The left-hand side equals 4.

Work out the right-hand side:

$$7 - 6 = 1$$

$$1 + 2 = 3$$

The right-hand side equals 3.

$$(2 + 7) - 5 > (7 - 6) + 2$$

## Questions B

Write the correct sign between these expressions.

1.  $(2 + 5) - 3 \square (7 - 2) + 5$
2.  $(12 - 7) + 13 \square (2 + 3) - 4$
3.  $(2 + 3) + 3 \square (6 + 4) + 5$
4.  $(2 + 8) - 5 \square (4 + 13) + 1$
5.  $(6 + 5) - 4 \square (8 + 5) - 4$
6.  $(6 - 5) + 11 \square (1 + 10) + 1$

### Hints / Notes

Work out the value of each side first. Remember BIDMAS, work out the inside of the brackets first.

*Trouble with this? Jump to: **Greater Than and Less than Notation** (p. 195); **BIDMAS** (p. 60)*

There are three more useful symbols:

### Key point

Greater than or equal to  $\geq$

Less than or equal to  $\leq$

Not equal to  $\neq$

We can write an expression using symbols to match a statement written in words. We use the letter, representing the item, then we choose the correct symbol for the statement.

### Example 29.3 — Write the symbol to match the expression.

The number of cats (c) is less than or equal to six.  $c \leq 6$

The number of bees (b) is not equal to 100.  $b \neq 100$

## Questions C

Write the symbol to match the expression.

1. The number of houses (h), is greater than four.
2. The number of stars (s), is less than twenty.



3. The number of trains (t), is greater than or equal to one.



4. The number of ice cubes (i), is less than or equal to five.



5. The number of sheep (s), is equal to eight.

6. The number of cars (c), is not equal to three.

7. The number of pens (p), is greater than or equal to five.

8. The number of poppies (p), is equal to three.



9. The number of votes (v), is not equal to fifty.

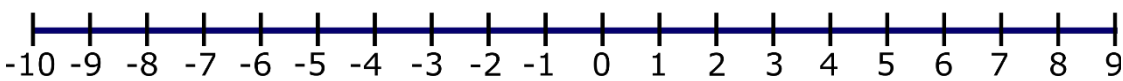
10. The number of horses (h), is less than or equal to seven.

## Chapter 30

# Ordering Integers

### Grade 1

On the number line, the number 6 is to the right of number 5 therefore 6 is larger than 5. The number  $-1$  (negative one) is to the right of  $-2$  (negative two) therefore  $-1$  is greater than  $-2$ .



### Questions A

Which of the two numbers is the largest?

- |                    |                     |
|--------------------|---------------------|
| 1. 5 and $-3$      | 2. $-2$ and $-1$    |
| 3. $-2$ and 7      | 4. $-4$ and 6       |
| 5. 2 and $-1$      | 6. 2 and $-10$      |
| 7. 1 and $-3$      | 8. $-1$ and 7       |
| 9. $-5$ and $-2$   | 10. $-15$ and $-12$ |
| 11. $-7$ and $-10$ | 12. 17 and $-3$     |

### Hints / Notes

If it's on the right of the number line, it's bigger! Did you notice that some questions had numbers that were not on the number line? Could you still work out the answer?

*Trouble with this? Jump to: **Greater Than and Less than Notation** (p. 195)*

### Example 30.1

Put these numbers in order, smallest to largest.

$-7, -8, 4, -3, 1$

The smallest number, the number most to the left on the number line is  $-8$ , the next number is  $-7$  and so on.

The order is:  $-8, -7, -3, 1, 4$

### Questions B

Put these numbers in order, smallest to largest.

1.5,  $-1$ , 6,  $-3$ , 0

**2.**  $-7, 2, -10, 1, 8$

**3.**  $-4, 3, 0, -9, 5$

**4.**  $7, -4, -6, 9, 0$

**5.**  $4, -1, -9, -3, 8$

**6.**  $10, -11, -6, -3, 2$

## Chapter 31

# Ordering Decimal Numbers

Grade 1

### 31.1 Method 1: Ordering by Making Whole Numbers.

We can change decimal numbers to whole numbers, then we can put them in order. We multiply by ten, then multiply by ten again, until all the numbers are whole numbers.

#### Example 31.1

Put these decimal numbers in order, smallest to largest:

2.51, 2.65, 2.57, 2.36, 2.6.

If we multiply the numbers by ten then by ten again, they all become *whole numbers*:

251, 265, 257, 236, 260

Put them in order: 236, 251, 257, 260, 265  
Write them as they were written originally.

The order is: 2.36, 2.51, 2.57, 2.6, 2.65

#### Questions A

Put these decimal numbers in order, smallest to largest.

1. 2.51, 2.65, 2.57, 2.36, 2.6
2. 7.215, 7.32, 7, 7.8, 7.351
3. 0.47, 0.465, 0.857, 0.855, 0.8
4. 5.25, 5.2, 6.78, 6.7, 6.25
5. 4.39, 4.93, 9.134, 9.34, 9.4
6. 6.1, 6.215, 6.665, 6.572, 6.21

#### Hints / Notes

Multiply by 10 several times until they are all whole numbers before putting them in order!

*Trouble with this? Jump to: **Multiplying Decimal Numbers by Powers of Ten** (p. 153)*

### 31.2 Method 2: Ordering by Place Value.

When comparing decimal numbers, *we start on the left* and compare the first digits. The larger digit means that this number is largest. If the digits are equal, we move right to compare the next digits. We continue to move right until we know which number is largest.

**Example 31.2** — Which number is greater? Use the symbols  $>$  or  $<$  in your answer.

5.62 and 4.81

Starting on the left we compare the Units:

5 is greater than 4.

$5.62 > 4.81$

7.406 and 7.45

The Units are equal, the Tenths are equal.

Move to the next column.

Compare the Hundredths: 0 is less than 5.

$7.406 < 7.45$

3.24 and 3.19

Starting on the left we compare the Units:

3 is equal to 3. The Units are equal. Move to the next column.

Compare the Tenths: 2 is greater than 1.

$3.24 > 3.19$

2.8 and 2.84

The Units are equal, the Tenths are equal.

Compare the Hundredths: 2.8 has 0 Hundredths. 0 is less than 4.

$2.8 < 2.84$

## Questions B

Which number is greater? Use the symbols  $>$  or  $<$  in your answer.

1. 5.3 and 7.8

3. 6.94 and 6.42

5. 1.809 and 1.89

7. 3.883 and 3.874

9. 14.58 and 14.584

2. 5.62 and 5.18

4. 11.8 and 12.84

6. 2.56 and 2.58

8. 7.951 and 7.952

### Hints / Notes

Remember  $>$  means greater than and  $<$  means less than. For example,  $8 > 6$  (8 is greater than 6).

8 is pointing to 6 saying 'you're small!'

*Trouble with this? Jump to: **Greater Than and Less than Notation** (p. 195); **Ordering by Making Whole Numbers** (p. 200)*

Sometimes we have many numbers to compare:

**Example 31.3** — Put these decimal numbers in order, smallest to largest.

2.51, 2.65, 2.57, 2.36, 2.6

The unit digits are the same so we compare the tenths:

2.51, 2.65, 2.57, 2.36, 2.6

2.82, 2.855, 2.17, 0.86, 2.8

Compare the unit digits:

2.82, 2.855, 2.17, 0.86, 2.8

2.36 is the smallest, followed by numbers starting with 2.5, then numbers starting with 2.6.

Numbers starting with 2.5: compare the hundredths for 2.51 and 2.57. 2.51 is the smallest, followed by 2.57.

Numbers starting with 2.6: compare 2.65 and 2.6. 2.6 has zero hundredths. 2.6 is the smallest, followed by 2.65.

The order is: 2.36, 2.51, 2.57, 2.6, 2.65

0.86 is the smallest number, followed by numbers starting with 2.

Compare the tenths: 2.82, 2.855, 2.17 and 2.8. 2.17 is the next smallest number, followed by numbers starting with 2.8.

Compare the hundredths: 2.82, 2.855 and 2.8. 2.8 has zero hundredths. 2.8 is the next smallest number, followed by 2.82 and 2.855.

The order is: 0.86, 2.17, 2.8, 2.82, 2.855

### Questions C

Put these decimal numbers in order, smallest to largest.

1. 0.61, 0.16, 0.6, 0.06, 1.06
2. 0.52, 0.255, 0.522, 0.5, 0.525
3. 0.82, 0.288, 0.832, 0.8, 0.838
4. 0.73, 0.7, 7.35, 0.705, 0.751
5. 0.312, 0.322, 0.31, 0.331, 0.3
6. 0.412, 0.49, 0.424, 0.4, 0.04

Trouble with this? Jump to: **Ordering Decimal Numbers by Making Whole Numbers** (p. 200)

## Chapter 32

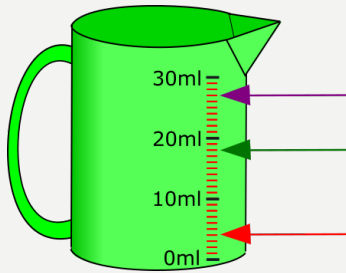
# Reading Scales on Measuring Instruments

### Grade 1

To read scales on an instrument, count the ticks from the first number to the next number, then you can work out how much one tick is worth. For example, if there are 10 ticks from 20ml to 30ml then each tick is worth 1ml. If instead, there are two ticks from 20ml to 30ml then each tick is worth 5ml.

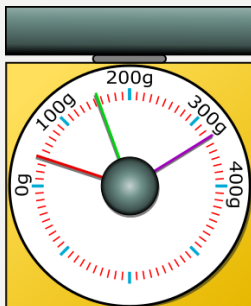
#### Example 32.1

What numbers are the pointers pointing to?



There are ten ticks between 0ml and 10ml. Each tick is 1ml.

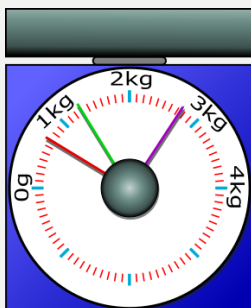
The arrows point to 5ml, 18ml and 27ml.



There are ten ticks between 0g and 100g. Each tick is 10g.

The green pointer is between 150g and 160g, it is 155g.

The pointers point to 40g, 155g and 330g.

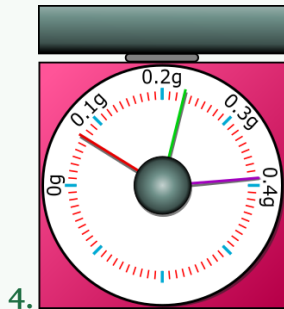
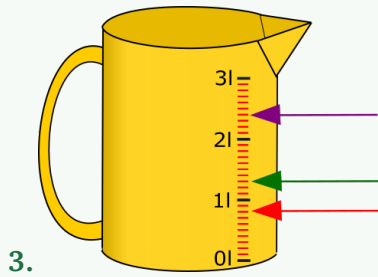
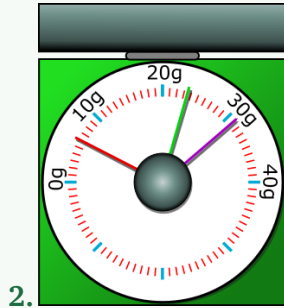
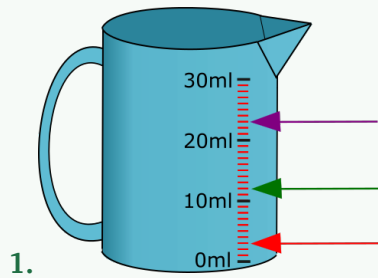


There are ten ticks between 0kg and 1kg. Each tick is 0.1kg.

The pointers point to 0.7g, 1.3g and 2.75g.

### Questions

What numbers are the pointers pointing to?



**Hints / Notes**

Figure out how much each dash is worth, then count the dashes!

*Trouble with this? Jump to: **Number Lines** (p. 209)*

## Chapter 33

# Time

### Grade 1

When we tell the time digitally, we may say “Its four forty”, when we say an analogue time, we may say “Its twenty to five”.

If the big hand is on the right it is *past* the hour. From the 12 we count up in 5-minute intervals as we jump from number to number clockwise. If the big hand is on the left it is *to* the hour. From the 12 we count up in 5-minute intervals as we jump anticlockwise.

To use a twenty-four-hour clock, we write the time in the afternoon or evening by adding twelve hours.

### Example 33.1

The analogue time is shown first, then the digital time.

(*am* – morning, *pm* – afternoon/evening).



Three O'clock

03:00 (am) or 15:00 (pm)



Half past five

05:30 (am) or 17:30 (pm)



Quarter past 12

00:15 (am) or 12:15 (pm)



Quarter to 11

10:45 (am) or 22:45 (pm)



Five past nine

09:05 (am) or 21:05 (pm)



Twenty-five past one

01:25 (am) or 13:25 (pm)



Ten to four

03:50 (am) or 15:50 (pm)



Twenty-two minutes to nine

08:38 (am) or 20:38 (pm)

### Questions A

Write the analogue and digital times for these clocks.



#### Hints / Notes

Remember, there are 365 days in a year, or more precisely  $365\frac{1}{4}$ . There are 52 weeks or 12 months.

### Questions B

Add hands to these clocks to show the correct time.



1. Twenty minutes to nine



2. 08:45



3. Ten minutes past seven



4. 13:30



5. Five minutes to five



6. 20:15



7.

Twenty-five minutes to two



8.

04:17



9.

Eleven minutes to one

## Chapter 34

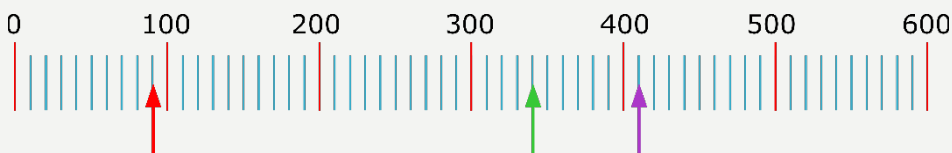
# Number Lines

### Grade 1

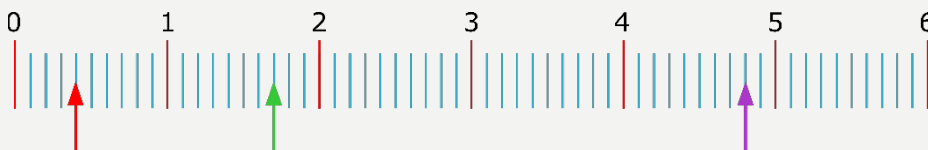
To find numbers on a number line, count the ticks from the first number to the second number, then you can work out how much one tick is worth.

#### Example 34.1

Look at the number lines below. What numbers are the arrows pointing to?



There are 10 ticks from 0 to 100 so each tick is worth  $100 \div 10 = 10$ . The red and green arrows are pointing to 90 and 340. The purple arrow is pointing to 410.



There are 10 ticks from 0 to 1 so each tick is worth  $1 \div 10 = 0.1$ . The red and green arrows are pointing to 0.4 and 1.7. The purple arrow is pointing to 4.8.



There are 10 ticks from 0 to 4 so each tick is worth  $4 \div 10 = 0.4$ . The red, green and purple arrows are pointing to 1.6, 12.4 and 21.2.



There are 5 ticks from 0 to 1 so each tick is worth  $1 \div 5 = 0.2$ . The red and green arrows are pointing to 0.2 and 2.8. The purple arrow is half way between 3.6 and 3.8 so the purple arrow is pointing to 3.7.

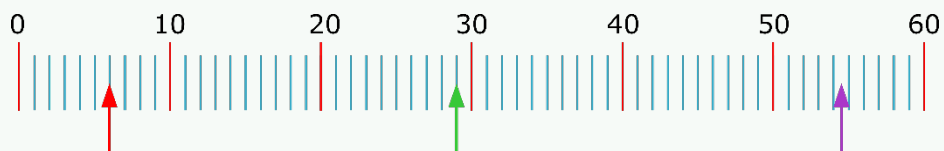
## Questions

Look at the number lines below. What numbers are the arrows pointing to?

1.



2.



3.



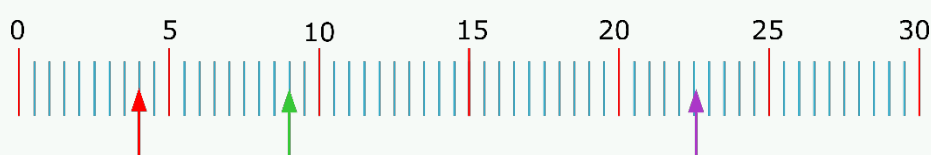
4.



5.



6.



#### Hints / Notes

Figure out how much each dash is worth, then count the dashes! If you have found the answer then check by counting along to see if you are right.

## Chapter 35

# Answers

---

### Place Value

#### Questions A P2

Larger	Smaller	Smaller
Larger	Larger	Larger
Smaller	Smaller	

#### Questions B P3

- a) 1799.7, b) 1799.7, c) 0.857  
a) 1.9435, b) 194.35, c) 84.5  
a) 303.03 b) 30.303 c) 81.9  
a) 485.06 b) 48.506 c) 0.0614  
a) 3.0858 b) 30.858 c) 83.4  
a) 190.8 b) 19.08 c) 159

### Rounding to the Nearest Whole Number P5

#### Questions A P6

7	8	10
29	76	14
16	3	3
6	12	70
28	19	11
43	84	6

#### Questions B P8

7.3	18.5	10.5
32.5	6.2	87.3
15.7	43.8	33.7
51.2	12.5	36.7
7.5	0.5	0.4
21.3	13.6	41.9

#### Questions C P9

7.37	28.16	10.49
18.45	6.81	62.25
35.61	25.31	3.74
3.46	72.45	12.41
7.55	57.25	10.36
10.36	0.99	40.50

## Questions D P10

9	8.9	8.94
7	7.4	7.39
11	11.4	11.37
6	5.9	5.86
14	13.7	13.68
3	3.0	2.96
4	3.6	3.65
12	12.4	12.37
4	3.7	3.71
3	2.6	2.57
7	7.0	6.99
1	0.6	0.60

**Rounding to One Significant Figure**

## Questions P11

8000	0.005
5000	60
700	200
50	4
4	0.03
6	0.08
200	0.00004
400	50000
0.03	300

**Rounding to Two Significant Figures**

## Questions P13

920	650	420
0.024	740	0.17
26	0.00099	3.9
790	6.4	49
230	4.3	710
0.00081	0.054	4000
0.0033	4100	

### Estimating Using One Significant Figure

#### Questions P15

$$90 \times 30 = 2700$$

$$\frac{600}{3} = 200$$

$$300 \times 6 = 1800$$

$$\frac{900}{30} = 30$$

$$3 \times 30 = 90$$

$$4 \times (70 - 10) = 240$$

$$800 \div 2 = 400$$

$$\frac{80 \times 2}{4} = 40$$

$$10 \div 20 = 0.5$$

$$\frac{6 \times 2}{40} = 0.3$$

$$10 \div 5 = 2$$

$$0.9 \times 90 = 81$$

### Ordering Negative Numbers

#### Questions P16

$$-9, -5, 0, 3, 8$$

$$-10, -6, -4, 9, 12$$

$$-15, -7, 0, 11, 13$$

$$-18, -12, -2, 7, 25$$

$$-21, -13, 41, 57, 73$$

$$-722, -175, 40, 119, 631$$

$$-6.2, -5.7, -4.8, 1.2, 9.3$$

$$-2, -1.8, 0, 2.5, 7.6$$

### Adding and Subtracting Negative Numbers

#### Questions A P18

2	6
8	17
-5	15
5	-1
0	2

## Questions B P19

-2	0
-4	-3
-2	-10
-5	-2
-5	-8
-1	-10

## Questions C P21

-3	-9	-2
-14	-8	-11
-5	-11	-7
-14	-2	-17
-6	-10	-8
-60	-17	-6

## Questions D P22

7	2
11	4
3	5
19	0
23	16

**Negative Numbers**  
Mixed Questions P23

-2	-8
7	-17
2	-3
-13	9
10	-17
-1	4
11	-11
1	-11
-17	14
-3	0

### Multiplying and Dividing with Negative Numbers

#### Questions A P24

15	-42
24	32
-40	-6
36	70

#### Questions B P25

6	-4
4	3
-2	-1
3	5

#### Mixed Questions P25

56	-99
6	-9
-48	1
-5	-70

### Square Numbers and Square Roots

#### Questions A P26

121	256	144
289	169	324
169	361	225
400		

#### Questions B P27

10.24	2.25	16
784	10609	250000

**Square Roots**

Questions P28

2	4	6	8
9	12	7	10

**Cube Numbers and Cube Roots**

Questions A P29

27	64	125
216	512	1000

Questions B P30

15.625	8000	-216
54.872	1000000	0.125

**Cube Roots**

Questions P31

10	3	5
1	2	8
-2	6	

**Indices and Roots**

Questions P33

256	1000000	243
390625	1	216
2	5	3
3		

**Common Indices or Powers**

Questions P34

1	1	17
1	1	1
35	1	

**Multiplying and Dividing Numbers with Indices**

Questions A P36

$2^7$	$2^2$	$5^8$
$6^8$	$3^8$	$8^7$
$8^{12}$	$4^{16}$	$2^{11}$
$6^2$	$5^1$	$7^2$
9	$8^6$	$3^{10}$
$4^3$	$2^4$	$9^9$

## Questions B P37

$5^8$	$3^{30}$	$3^6$
$4^{16}$	$10^6$	$3^8$
$6^6$	$7^{-8}$	$5^{-24}$
$3^8$	$5^{10}$	$3^2$

**Reciprocals**

## Questions P38

$\frac{1}{3}$	$\frac{7}{2}$	$\frac{1}{7}$
$\frac{8}{3}$	$\frac{1}{9}$	$\frac{3}{11}$
$\frac{1}{2}$	$\frac{10}{9}$	6
$\frac{7}{8}$	5	$\frac{8}{17}$
10	$\frac{5}{18}$	

**Divisibility**

## Questions P40

	Divides by:
216	2,3,4,6,9
114	2,3,6
775	5
540	2,3,4,5,6,9,10
198	2,3,6,9
243	3,9
831	3
528	2,3,4,6
258	2,3,6
1328	2,4
1255	5
9132	2,3,4,6

**Standard Index Form**

## Questions A P42

$8.3 \times 10^4$	$2 \times 10^5$
$7.4 \times 10^3$	$7 \times 10^3$
$5.62 \times 10^5$	$9.87 \times 10^3$
$4.583 \times 10^2$	$6.5426 \times 10^3$
$9.438 \times 10^1$	

## Questions B P43

$4.1 \times 10^{-3}$	$6.1 \times 10^{-2}$
$3.15 \times 10^{-3}$	$5.4 \times 10^{-3}$
$2.3 \times 10^{-4}$	$7.3 \times 10^{-3}$
$6.1 \times 10^{-6}$	$9 \times 10^{-5}$
$5.2 \times 10^{-3}$	

## Questions C P45

1200	20000
8000000	4300
64000	8950000
0.00436	0.000649
0.0000712	

## Questions D P47

$6 \times 10^5$	$8 \times 10^7$
$5 \times 10^8$	$2 \times 10^1$
$3.6 \times 10^8$	$3 \times 10^5$
$3.56 \times 10^3$	$6.6 \times 10^3$

## Questions E P48

$1.272 \times 10^7$	$1.104 \times 10^{-6}$
$5 \times 10^7$	$4 \times 10^2$

## Questions F P49

$5.54 \times 10^{13}$	$7.9 \times 10^{14}$
$9.712 \times 10^{18}$	$4.318 \times 10^{23}$

**Prime Numbers**

## Questions A P50

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47,  
53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

## Questions B P51

	2	3	5	7	11	13	17
323	N	N	N	N	N	N	Y
377	N	N	N	N	N	Y	N
241	N	N	N	N	N	N	N
143	N	N	N	N	Y	Y	N

241 could be a prime number as none of these numbers divide into it.

### Factors of a Number

#### Questions P53

1 and 24, 2 and 12, 3 and 8, 4 and 6.

1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6.

1 and 32, 2 and 16, 4 and 8.

1 and 40, 2 and 20, 4 and 10, 5 and 8.

1 and 15, 3 and 5.

1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8.

1 and 52, 2 and 26, 4 and 13.

1 and 60, 2 and 30, 3 and 20, 4 and 15, 5 and 12, 6 and 10.

1 and 28, 2 and 14, 4 and 7.

1 and 42, 2 and 21, 3 and 14, 6 and 7.

### Prime Factors of a Number

#### Questions P53

1,2,3,5,6,10,15,30. Primes 2, 3, 5.

1,2,3,4,6,8,12,16,24,48. Primes 2,3.

1,2,5,7,10,14,35,70. Primes 2,5,7.

1,2,4,8,16,32. Prime 2.

1,2,4,7,8,14,28,56. Primes 2,7.

1,2,3,4,6,8,12,24. Primes 2,3.

1,2,3,6,11,22,33,66. Primes 2,3,11.

1,5,7,35. Primes 5,7.

### Prime Factors using Factor Trees

#### Questions P55

$$2^2 \times 3^3$$

$$2 \times 3^2 \times 5^2$$

$$2^3 \times 17$$

$$2^3 \times 3^2 \times 7$$

$$2 \times 3^2 \times 5$$

$$2^3 \times 3^3$$

$$2 \times 3 \times 13$$

$$11 \times 13$$

**Highest Common Factor (HCF)**

Questions P56

4	15
4	8
12	28
15	4

**Multiples of a Number**

Questions P57

4,8,12,16,20.  
 5,10,15,20,25.  
 8,16,24,32,40.  
 10,20,30,40,50.  
 9,18,27,36,45.  
 7,14,21,28,35.  
 11,22,33,44,55.  
 15,30,45,60,75.

**Lowest Common Multiple (LCM)**

Questions P57

20	21
18	12
35	40
24	45
12	72

**HCF and LCM from Prime Factors**

Questions P59

14, 140	3,585
6,132	15,90
6, 72	15, 90
12, 144	4,144
36, 5040	33, 462
15, 630	12, 240

**Doubling and Halving**

## Questions P61

$7 \times 6 = 42$	$9 \times 10 = 90$
$16 \times 10 = 160$	$8 \times 12 = 96$
$8 \times 9 = 72$	$10 \times 7 = 70$
$6 \times 8 = 48$	$4 \times 6 = 24$

**Multiplying with Decimal Numbers**

## Questions P63

2.4	0.6
5.6	3.2
2.7	0.4
0.08	0.48
0.81	55.8
4.14	1.96
0.7	1.74
2.1	4.88
3.04	3.33
0.108	43.2

**Dividing with Decimal Numbers**

## Questions P65

30	8
2	12
17	24
160	130
40	300
50	40

**Multiplying Decimal Numbers with Two or Three Digits**

## Questions P71

57.6	215.8
239.2	82.8
13.775	9.062
53.055	48.436

### Dividing Whole Numbers and Decimals by Single Digit Numbers

#### Questions A P73

4.9	2.9
2.44	7.1
20.2	0.67
1.29	6.4
133.7	0.107
0.923	6.41
2.51	1.62
2.715	16.15
1.88	2.485
1.48	0.65
4.75	

#### Questions B P74

$26.6 \div 8 = 3.325$	$230 \div 7 = 32.86$
$50 \div 3 = 16.67$	$97 \div 8 = 12.125$
$434.6 \div 5 = 86.92$	$691.2 \div 3 = 230.4$
$315.4 \div 11 = 28.67$	$678.6 \div 12 = 56.55$

### Dividing Whole Numbers and Decimals by Two Digit Numbers

#### Questions P77

24	17
31	23
41	14
36	42
15.4	24.1
19.6	173
31.8	16.25
54.2	

### Problem Solving

#### Questions P79

$$1820 \times 7 = 12740\text{kg}$$

$$280 \times 2.1 = 588\text{g}$$

$$201 \div 15 = 13.4\text{g}$$

$$18.12 \div 12 = \text{£}1.51$$

$$7.5 \times 14 = 105\text{km}$$

$$517.5 \div 45 = 11.5 \text{ days.}$$

**BIDMAS**

## Questions A P81

22	30
4	24
14	3
25	32
1.7	21
13	30
17	35
10	4
6	4
46	46
15	19
25	3

## Questions B P82

13	99
54	24
80	20
55	25
10	32
55	39
28	12
26	63

**Inverse Operations**

## Questions A P83

$$13-5=8, 13-8=5$$

$$8-6=2, 8-2=6$$

$$10-4=6, 10-6=4$$

$$17-12=5, 17-5=12$$

$$15-7=8, 15-8=7$$

$$22-18=4, 22-4=18$$

$$11-9=2, 11-2=9$$

$$16-13=3, 16-3=13$$

## Questions B P83

$$24 \div 4 = 6, 24 \div 6 = 4$$

$$36 \div 12 = 3, 36 \div 3 = 12$$

$$42 \div 7 = 6, 42 \div 6 = 7$$

$$72 \div 8 = 9, 72 \div 9 = 8$$

$$40 \div 5 = 8, 40 \div 8 = 5$$

$$21 \div 7 = 3, 21 \div 3 = 7$$

$$45 \div 9 = 5, 45 \div 5 = 9$$

$$77 \div 11 = 7, 77 \div 7 = 11$$

## Questions C P84

$6 \times 5 \neq 40$	Not Correct
$9 \times 8 \neq 63$	Not Correct
$9 \times 8 = 72$	Correct
$12 \times 6 = 72$	Correct
$7 \times 6 \neq 56$	Not Correct
$5 \times 4 \neq 25$	Not Correct
$31 \times 3 = 93$	Correct
$6 \times 8 = 48$	Correct

**Fractions of a Whole**

## Questions A P85

$$\frac{7}{8}$$

$$\frac{7}{10}$$

$$\frac{5}{9}$$

$$\frac{1}{7}$$

$$\frac{2}{9}$$

$$\frac{3}{11}$$

$$\frac{2}{5}$$

$$\frac{7}{12}$$

$$\frac{3}{4}$$

$$\frac{2}{3}$$

$$\frac{3}{8}$$

$$\frac{1}{8}$$

## Questions B P86

$$\frac{3}{4} + \frac{2}{5} + \frac{7}{10}$$

$$\frac{2}{3} + \frac{4}{5} + \frac{2}{9}$$

**Fractions of Shapes**

Questions A P88

$$\frac{1}{3} + \frac{1}{2} + \frac{4}{9} + \frac{3}{5}$$

$$\frac{1}{6} + \frac{6}{7} + \frac{4}{15} + \frac{5}{16}$$

Questions B P89

$$\frac{3}{5} + \frac{5}{8}$$

$$\frac{3}{4} + \frac{3}{9}$$

$$\frac{4}{7} + \frac{3}{5}$$

**Fractions of a Quantity**

Questions A P90

$$5 + 9 + 5 + 4 + 3$$

$$6 + 8 + 5 + 3 + 7$$

Questions B P92

$$10 + 27 + 15 + 20$$

$$63 + 9 + 12 + 25$$

Questions C P93

Theo read 12 messages.

The smallest plant is 27cm.

She spends £6 on sweets and music.

Lewis has 16 elephants.

Harry cycled 9 miles.

13 million people are ex-smokers.

**Equivalent Fractions**

Questions A P95

$$\begin{aligned} \frac{1}{4} &= \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} \\ \frac{1}{3} &= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} \\ \frac{1}{5} &= \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} \\ \frac{1}{10} &= \frac{2}{20} = \frac{3}{30} = \frac{4}{40} = \frac{5}{50} \\ \frac{3}{4} &= \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} \end{aligned}$$

Questions B P95

$$\begin{aligned} \frac{1}{3} &= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} \\ \frac{2}{3} &= \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} \\ \frac{6}{9} &= \frac{8}{12} = \frac{10}{15} \end{aligned}$$

2)

$$\begin{aligned} \text{a) } \frac{3}{4} &= \frac{6}{8} = \frac{9}{12} = \frac{15}{20} \\ \text{b) } \frac{5}{8} &= \frac{10}{16} \end{aligned}$$

3)

$$\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{6}{30}$$

Questions C P97

12	12	20
27	32	54
63	32	2
3	4	4
6	15	16
15	5	6
3	4	5
8	7	7

**Simplifying Fractions by Cancelling Down**

Questions A P99

$\frac{1}{3}$	$\frac{4}{5}$	$\frac{5}{6}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{5}$	$\frac{8}{9}$
$\frac{3}{4}$	$\frac{7}{5}$	$\frac{5}{7}$
$\frac{1}{4}$	$\frac{7}{12}$	$\frac{2}{7}$
$\frac{1}{5}$	$\frac{3}{5}$	$\frac{7}{7}$
$\frac{16}{25}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{29}{30}$	$\frac{2}{3}$	$\frac{1}{3}$
$\frac{9}{10}$	$\frac{1}{4}$	$\frac{1}{3}$

Questions B P100

$$\frac{1}{2} = \frac{28}{56} = \frac{9}{18}$$

$$\frac{1}{4} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{10}{30}$$

$$\frac{3}{4} = \frac{54}{72}$$

$$\frac{1}{5} = \frac{12}{60}$$

**Changing Mixed Numbers to Improper Fractions**

Questions P103

$\frac{6}{5}$	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{11}{4}$	$\frac{22}{7}$	$\frac{11}{5}$
$\frac{7}{2}$	$\frac{11}{10}$	$\frac{23}{3}$
$\frac{21}{8}$	$\frac{97}{12}$	$\frac{21}{4}$

**Changing Improper Fractions to Mixed Numbers**

Questions P105

$4\frac{1}{4}$	$2\frac{3}{5}$	$6\frac{4}{5}$
$2\frac{5}{6}$	$3\frac{1}{3}$	$1\frac{2}{5}$
$4\frac{3}{4}$	$3\frac{1}{7}$	$3\frac{1}{8}$
$5\frac{1}{8}$	$2\frac{1}{11}$	$6\frac{3}{7}$

**Ordering Fractions by Comparing to One Half or One Whole**

Questions P106

$\frac{7}{12}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{5}$
$\frac{4}{5}$	$\frac{6}{7}$
$\frac{5}{7}$	$\frac{5}{6}$
$\frac{7}{10}$	

**Ordering Fractions by Using Equivalent Fractions**

Questions P108

$$\frac{1}{10}, \frac{1}{5}, \frac{4}{10}, \frac{3}{5}, \frac{7}{10}, \frac{9}{10}$$

$$\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}$$

$$\frac{1}{10}, \frac{3}{20}, \frac{2}{5}, \frac{9}{20}, \frac{7}{10}, \frac{4}{5}$$

$$\frac{1}{16}, \frac{5}{16}, \frac{3}{8}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}$$

$$\frac{1}{9}, \frac{7}{27}, \frac{11}{27}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}$$

$$\frac{1}{8}, \frac{8}{24}, \frac{5}{12}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}$$

**Adding and Subtracting Fractions**

Questions A P108

$\frac{7}{8}$	$\frac{5}{9}$	$\frac{2}{5}$
$\frac{6}{11}$	$\frac{4}{7}$	$\frac{10}{15}$
$\frac{2}{5}$	$\frac{6}{11}$	

Questions B P110

$$\begin{array}{ccc} \frac{13}{14} & \frac{2}{5} & \frac{1}{5} \\ 1\frac{5}{32} & 1 & \frac{17}{42} \\ 1\frac{1}{9} & \frac{14}{33} & \frac{5}{12} \\ \frac{34}{45} & & \end{array}$$

## Questions C P111

$$\begin{array}{ccc} \frac{22}{35} & \frac{17}{45} & \frac{11}{20} \\ 1\frac{1}{18} & 1\frac{13}{30} & \frac{17}{24} \\ 2\frac{19}{22} & 1\frac{5}{39} & \end{array}$$

**Adding and Subtracting Mixed Numbers**

## Questions P113

$$\begin{array}{ccc} 4\frac{5}{28} & \frac{9}{10} & 8\frac{1}{12} \\ 8\frac{11}{12} & 1\frac{7}{8} & \frac{8}{9} \\ 3\frac{9}{14} & 5\frac{1}{15} & \end{array}$$

**Multiplying Fractions**

## Questions P115

$$\begin{array}{ccc} \frac{1}{4} & \frac{9}{14} & \frac{1}{10} \\ \frac{6}{35} & \frac{6}{25} & \frac{1}{12} \\ \frac{27}{80} & 2\frac{1}{5} & \end{array}$$

**Multiplying Mixed Numbers**

## Questions P117

$$\begin{array}{ccc} 12\frac{1}{4} & 16\frac{1}{3} & 7\frac{4}{5} \\ 7\frac{3}{5} & 6\frac{4}{5} & 4 \\ 8\frac{4}{9} & 9 & \end{array}$$

**Multiplying Fractions and Whole Numbers**

## Questions P118

$$\begin{array}{ccc} 6\frac{3}{5} & 2\frac{1}{2} & \frac{6}{13} \\ 5\frac{5}{11} & 4\frac{4}{7} & 5\frac{5}{6} \\ 2\frac{2}{3} & 1\frac{7}{8} & \end{array}$$

**Dividing Fractions**

## Questions P120

$$\begin{array}{ccc} 1\frac{1}{2} & \frac{27}{28} & 2\frac{1}{3} \\ 1\frac{3}{5} & \frac{1}{7} & 1\frac{29}{55} \\ \frac{8}{9} & 1\frac{2}{3} & \end{array}$$

**Dividing Mixed Numbers**

## Questions P122

6	$1\frac{14}{31}$	$2\frac{4}{25}$
$\frac{2}{11}$	$\frac{32}{63}$	$1\frac{8}{19}$
$\frac{5}{16}$	$\frac{11}{28}$	

**Dividing Fractions and Whole Numbers**

Questions P124

$\frac{5}{42}$	$\frac{7}{33}$	$10\frac{1}{2}$
12	$\frac{4}{45}$	$\frac{1}{6}$
32	18	

**Percentages**

Questions P125

40%	75%	28%
67%	35%	94%
43%	1%	

**Decimal Numbers and Percentages**

Questions A P126

31%	45%	50%
65%	64%	21%
7%	93%	275%
125%		

Questions B P127

0.33	0.17	0.2
0.23	0.75	0.78
0.5	0.55	0.66
0.08	0.06	0.97
0.8	1.13	

**Percentages and Fractions**

Questions P128

$\frac{9}{50}$	$\frac{1}{2}$	$\frac{9}{20}$
$\frac{4}{5}$	$\frac{21}{25}$	$\frac{3}{4}$
$\frac{3}{10}$	$\frac{1}{5}$	

**Fractions to Percentages**

Questions A P130

22%	28%	5%
30%	48%	60%
65%	24%	86%
95%		

## Questions B P131

60%	10%	25%
30%	40%	75%
70%	90%	80%
15%		

## Questions C P132

27.3%	26.9%	62.5%
93.8%	81.3%	75.6%
52.1%	53.6%	46.7%
80.6%		

## Questions D P134

40%	84%	60%
46.7%	80%	22.2%
64%	42.2%	

**Decimal Numbers and Fractions**

## Questions P136

$\frac{4}{5}$	$\frac{3}{20}$	$\frac{73}{100}$
$\frac{3}{10}$	$\frac{23}{25}$	$\frac{6}{25}$
$\frac{3}{4}$	$\frac{171}{200}$	$1\frac{2}{5}$
$2\frac{17}{20}$		

**Fractions to Decimal Numbers**

## Questions A P137

0.1	0.35	0.7
0.32	0.6	0.65
0.44	0.8	

## Questions B P139

1.3	8.68	4.9
7.2	2.31	4.48
3.4	6.65	

## Questions C P140

0.75	0.429	0.833
0.222	0.8	0.375
0.625	0.273	

## Questions D P141

0.28	0.43	0.45
0.59	0.94	0.47
0.28	1.2	0.86
2.46		

**Percentages of Shapes**

## Questions P144

25%	50%	25%
16.7%	75%	40%

**Percentages of a Quantity Non-Calculator Method**

## Questions A P146

£40	£5.86	£9.00
2.7kg	£3.26	4.87ml
£6.05	3.4g	£1.21
8.9m		

## Questions B P147

£60	7.4ml	£27.20
18 bottles	£32.00	4.8km
55.8kg	35.1cm	60ml
£22.68		

## Questions C P148

£0.57	£3.24	14.5kg
£315.00	£28.00	6.3m
153g	35.75L	82.65mm
£19.85		

## Questions D P150

51.7m	438.6g	£45.60
442L	60.9km	151.2mm
£2.29	£5.70	97.50
£208.00	13.2kg	43.68m

### Percentage Increase and Decrease Non-Calculator Method

Questions P152

£540	57.85
64.4kg	58.52m
79.5km	£10.14
£89.54	£468
1078.74g	9.1mm
1022.35ml	£632.40

### Percentage Increase and Decrease Problems

Questions P154

£20.80	504 sheep
£1776	46 apples
£62300	27.28 miles
£52.00	£9440
£117	£22660
£38.25	£30.80

### Percentages of a Quantity Calculator Method

Questions P157

£31.28	723.9g
£3.74	17.82kg
£15.70	£55.10
0.9564m	£1.94
2.94cm	6.525L

### Percentage Increase Calculator Method

Questions P157

£88.20	6.075L
12.936cm	110 chips
£571.96	£64.79
£9.52	£892.50
60.5kg	56.1g

**Percentage Decrease Calculator Method**

Questions P160

£43.00	£27.90
168kg	3115 crops
£176.66	£72.60
217.5ml	5.84ml
46.2kg	384 seeds

**Percentage Increase and Decrease Problems – Calculator Method**

Questions P160

5850 poppies	£6.33
£345	£280
176 dresses	£220.80

**Repeating Percentage Increase and Decrease Calculator Method**

Questions P163

£42.30	£44.78
£40.56	£27.85
123.02g (2 d.p.)	32.58g (2 d.p.)
103.42cm (2 d.p.)	£22.18
£9.90	£245.86
139.92kg (2.d.p.)	0.6cm

**Repeating Percentage Increase or Decrease Problems**

Questions P163

£159040.18	£1.74
18216.84	441.8g
£274.30	21kg
131 buses	£20523.38
20.2km	74.4cm
281ml	£327.68

**Percentage Profit and Percentage Loss**

Questions P166

15%	7.7%
20.9%	26.2%
28.6%	30.9%
35.9%	1100%
75%	93.5%

**Mixed Percentage Questions**

## Questions P167

56.7%	66.7%
65%	58%
£172.50	£12.00
2.675m	£26.40
36 children	£92741.93
22.5%	79%

**Reverse Percentages**

## Questions P170

53.6kg	35 insects
£304.76	62
78.3ml	4545.45kg
£54.22	200 cards
37°C	£2.50

**Simple Interest**

## Questions P172

£235.20, 1075.20	£2550, £9350
£30,000, £80,000	£2322, £10,062
£490, £7490	£418, £1368

**Understanding Ratio**

## Questions A P174

3:7	4:11
5:8	1:5
13:27	4:5
5:12	2:1
8:5	7:13
1:4	2:3
3:4	5:6

## Questions B P175

4:5	2:3
17:6	5:2

**Ratio and Fractions**

## Questions P177

- a)  $3 : 2$ , b)  $\frac{3}{5}$ , c)  $\frac{3}{2}$   
a)  $4 : 5$ , b)  $4 : 9$ , c)  $\frac{4}{5}$   
a)  $\frac{5}{12}$ , b)  $\frac{12}{17}$ , c)  $12 : 5$   
a)  $2 : 5$ , b)  $\frac{3}{5}$ , c)  $\frac{2}{3}$

### Ratio Problems

#### Questions A P179

- 6 girls  
72 ordered coffee  
40 children  
20 students  
12cm  
10 crew  
24 cakes  
18 messages  
5 litres

#### Questions B P181

- Sunni has £1.20  
There are 55 coaches  
200g of flour  
US 75, China 70  
75 were saved  
72cm

### Sharing in a Given Ratio

#### Questions P183

- |              |                   |
|--------------|-------------------|
| £80 and £320 | 60 and 80 marbles |
| 250m         | 120g and 150g     |
| 20 hours     | 420 voters        |
| 39 steps     | 78 flowers        |

### Ratio Questions with Algebra

#### Questions P186

Simone 16 Tobias 20

Length is 6cm

Mira 10 Kira 15

Radius is 7.5 cm

33 potatoes

Length is 4 cm

Petra has £13.50

Kate has 25 conkers

### Proportion

Questions P188

£3.40	£3.60
Butter 550g	36cm
£32.00	770 vehicles
6 minibuses	5 times
£45.60	70 snails
30 folders	£540

### Proportion Unitary Method

Questions P190

£376	95 square metres
£85	133.3g butter and 200g flour
25 cakes	£40
550g	120g butter and 53.3g flour
375 screws	12 more pages

### Exchange Rates

Questions A P192

\$120	£96	£80
£16.80	£10.88	€144
£215.38	£65	155.36
£21.74		

Questions B P194

It costs £8.45 in Dublin, Dublin is cheapest.	They cost £28.20 in Orlando. Orlando is cheapest.
It costs £23.08 in Tokyo. Tokyo is cheapest.	It costs \$1.41 NZD in Bristol. Christchurch is cheapest.
It costs £726.55 in Moscow. Moscow is cheapest.	It costs £11.63 in Cairo. £2.13 is the difference.

**Using a Calculator**

Questions P196

76.62	5.37
6.22	3.66
1.27	101.85
14.02	126.42
4.52	0.20

**Upper and Lower Bounds**

Questions P198

0.5g, 23.5g, 22.5g  
 5kg, 65kg, 55kg  
 0.05cm, 2.55cm, 2.45cm  
 50kg, 4850kg, 4750kg  
 2.5g, 67.5g, 62.5g  
 25miles, 275miles, 225miles  
 1cm, 59cm, 57cm  
 10g, 750g, 730g

**Upper and Lower Bounds**

Questions P198

- 
- 1)  $27.15 \leq b < 27.25$
  - 2)  $92.7 \leq p < 92.9$
  - 3)  $35 \leq s < 45$
  - 4)  $410 \leq h < 430$
  - 5)  $750 \leq t < 850$
  - 6)  $22.5 \leq f < 27.5$
  - 7)  $15 \leq c < 25$
  - 8)  $7.75 \leq b < 7.85$

**Place Value**

## Questions A P201

3000, 400, 90, 1	2000, 900
9000, 10, 5	9000, 200, 1
600, 30, 1	2000, 5
7000, 800, 30	900, 80, 8

## Questions B P202

1, 0.4, 0.05, 0.002	50, 6, 0.3, 0.02, 0.005
0.5, 0.01, 0.004	300, 20, 4, 0.5, 0.06
20, 5, 0.3, 0.01, 0.002	0.8, 0.004
600, 3, 0.09,	30, 0.01, 0.005

**Multiplying Whole Numbers by Powers of Ten**

## Questions P204

80	70	8920
960	3240	6270
250	30	8160
200	1060	9800
17080	54310	200
600	5400	3200
81600	20700	106400
980000	4000	8000
15000	39000	855000
227000		

**Dividing Whole Numbers by Powers of Ten**

## Questions P206

485	283	80
45	68.5	21.8
42	45.1	3
7	2.5	6.2
8.9	1.7	0.7
0.9	27.56	62.54
8.92	1.73	0.76
0.98	0.2	0.09
0.7	0.06	8.912
1.073	0.764	0.908
0.025	0.081	

### Multiplying Decimal Numbers by Powers of Ten

#### Questions P208

5	3	65
93	42.5	61.1
32.4	378	929
75.5	1.4	8.8
70.87	603.1	45.06
160.3	87	22
57.8	19.5	703.7
689.1	4250.6	1603
823.7	212.9	543
215	7070	6410
85506	16.3	

### Dividing Decimal Numbers by Powers of Ten

#### Questions P210

3.24	7.78	3.291
6.258	23.61	18.93
6.18	3.47	1.26
1.97	6.04	9.05
0.57	0.31	0.468
0.691	4.416	6.012
81.721	24.108	3.624
7.878	1.529	0.625
0.261	1.893	0.618
3.047	1.5426	1.8797
6.0124	0.9205	0.0527
0.0031		

### Rounding to the Nearest 10, 100 and 1000

#### Questions A P211

20	80
70	40
60	80
50	10
80	20
20	50

#### Questions B P212

200	400
600	600
400	300
100	900
500	1000
200	300

#### Questions C P213

3000	3000
7000	5000
8000	6000
8000	9000
1000	6000
4000	10000

**Adding Whole Numbers Mental Method**

Questions P214

68	98
150	140
91	103
733	410
1130	

**Adding Whole Numbers Written Method**

Questions P216

448	373
1146	828
621	942
1325	980
1641	

**Subtracting Whole Numbers Mental Method**

Questions A P218

27	18
28	28
49	55
66	44
348	128

Questions B P219

43	42	53
23	48	117
68	68	69

**Subtracting Whole Numbers Written Method**

Questions P221

244	323	380
478	213	561
427	444	89

**Adding Decimal Numbers Mental Method**

Questions P223

1.2	1.4	1.3
5.3	8.0	17.5
3.0	4.4	0.6
7.2	1.0	14.2

**Adding Decimal Numbers Written Method**

Questions P225

7.46	57.5	52.0
17.33	40.7	8.72
58.88	15.97	26.3

**Subtracting Decimal Numbers Mental Method**

Questions P227

0.7	8.1	4.1
2.3	0.9	3.6
6.5	1.7	4.8

**Subtracting Decimal Numbers Written Method**

Questions P230

5.13	61.7	8.3
4.07	19.1	1.87
32.5	2.19	0.41

**Multiplying Using the Times Tables or By Counting On**

Questions P231

25	20	18	21
36	72	35	56
32	8	45	24
44	12	48	54
72	12	28	36
36	28	63	81
30	40	44	77
6	9	40	64
24	48	14	22
42	49	18	108

**Multiplying Whole Numbers by Single Digits**

Questions A P233

180	210	280
180	240	350
1200	2400	3500
1200		

## Questions B P234

186	190	288
190	268	390
448	470	2924
1310	3432	1992

**Multiplying Whole Numbers by Two Digits Column Method**

## Questions P237

1472	1887	3690
3367	1872	1568
13328	11421	27572
22704		

**Dividing Using the Times Tables or By Counting On**

## Questions P237

1	9	2	6
7	10	11	4
6	5	3	2
4	4	6	8
3	8	1	10
5	2	3	7
9	11	8	7
3	12	5	8
2	8	3	7
3	4	4	12
8	6	12	4
8	12		

**Dividing Whole Numbers by Single Digits**

## Questions P240

34	21	24
181	79	56
144r1	113r2	131r2
85r4		

**Dividing Whole Numbers by Two Digits Chunking Method**

## Questions P242

12	45	43
25	28r1	27r2
36	52	

**Multiplying Decimal Numbers with One Digit**

## Questions A P243

1.5	4.9
6.3	1.2
3.2	1.2
0.08	5.4
0.2	2.4

## Questions B P244

0.15	0.28
0.54	0.4
0.18	0.64
0.96	0.33
0.45	0.72

**Negative Numbers on a Thermometer**

## Questions A P245

2°C	-2°C	3°C
0°C	-1°C	-3°C

## Questions B P247

6°C	6°C	5°C
8°C	11°C	8°C

**Factor Spiders**

## Questions P248

1,2,3,6,9,18	1,2,4,7,14,28
1,2,4,8,16,32	1,2,3,4,6,9,12,18,36
1,3,5,9,15,45	1,2,3,4,5,6,10,12,15,20,30,60

**Fraction Names**

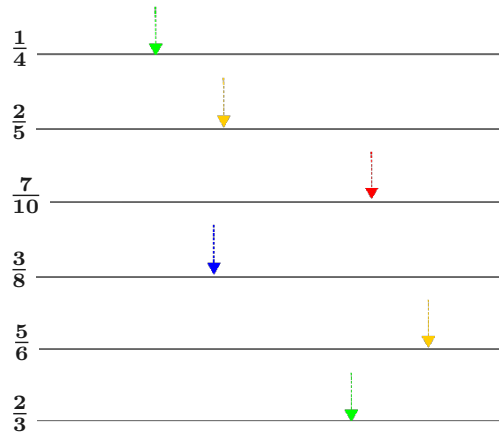
## Questions A P250

- five sixths,  $\frac{5}{6}$
- Two fifths,  $\frac{2}{5}$
- One third,  $\frac{1}{3}$
- Three eighths,  $\frac{3}{8}$
- Four sevenths,  $\frac{4}{7}$
- Five ninths,  $\frac{5}{9}$
- Five ninths,  $\frac{5}{9}$
- Five elevenths,  $\frac{5}{11}$

Questions B P251

- |                |                |
|----------------|----------------|
| $\frac{1}{5}$  | $\frac{1}{4}$  |
| $\frac{1}{3}$  | $\frac{4}{5}$  |
| $\frac{1}{10}$ | $\frac{7}{10}$ |

Questions C P252



**Ordering Fractions with the Same Denominator**

Questions P253

- |  |  |
|--|--|
| $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$ | $\frac{1}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ |
| $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$     | $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$ |
| $\frac{1}{9}, \frac{2}{9}, \frac{5}{9}, \frac{7}{9}$     |  |

**Adding Fractions with the Same Denominator**

Questions P255

$\frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{3}{7}$	$\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$
$\frac{4}{10} + \frac{2}{10} = \frac{6}{10}$	$\frac{3}{11} + \frac{2}{11} + \frac{1}{11} = \frac{6}{11}$
$\frac{1}{6} + \frac{1}{6} + \frac{2}{6} = \frac{4}{6}$	$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

**What is a Decimal?**

Questions P256

0.1	0.5
0.7	0.8
0.4	0.2
0.3	0.9
0.6	1.5

**Greater Than and Less Than Notation**

## Questions A P257

$5 < 6$	$6 > 2$
$11 > 2$	$15 = 15$
$9 = 9$	$9 < 10$
$4 < 7$	$7 > 1$
$33 > 5$	$5 < 12$
$93 > 9$	$0 < 2$

## Questions B P258

$$(2+5)-3 < (7-2)+5$$

$$(12-7)+13 > (2+3)-4$$

$$(2+3)+3 < (6+4)+5$$

$$(2+8)-5 < (4+13)+1$$

$$(6+5)-4 < (8+5)-4$$

$$(6-5)+11 = (1+10)+1$$

## Questions C P259

$h > 4$	$s < 20$
$t \geq 1$	$i \leq 5$
$s = 8$	$c \neq 3$
$p \geq 5$	$p = 3$
$v \neq 50$	$h \leq 7$

**Ordering Integers**

## Questions A P260

5	-1
7	6
2	2
1	7
-2	-12
-7	17

## Questions B P261

-3, -1, 0, 5, 6	-10, -7, 1, 2, 8
-9, -4, 0, 3, 5	-6, -4, 0, 7, 9
-9, -3, -1, 4, 8	-11, -6, -3, 2, 10

**Ordering Decimal Numbers**

## Questions A P262

2.36, 2.51, 2.57, 2.6, 2.65  
 7, 7.215, 7.32, 7.351, 7.8  
 0.465, 0.47, 0.8, 0.855, 0.857  
 5.2, 5.25, 6.25, 6.7, 6.78  
 4.39, 4.93, 9.134, 9.34, 9.4  
 6.1, 6.21, 6.215, 6.572, 6.665

## Questions B P264

$5.3 < 7.8$	$5.62 > 5.18$
$6.94 > 6.42$	$11.8 < 12.84$
$1.809 < 1.89$	$2.56 < 2.58$
$3.883 > 3.874$	$7.951 < 7.952$
$14.58 < 14.584$	

## Questions C P265

0.06, 0.16, 0.6, 0.61, 1.06  
 0.255, 0.5, 0.52, 0.522, 0.525  
 0.288, 0.8, 0.82, 0.832, 0.838  
 0.7, 0.705, 0.73, 0.751, 7.35  
 0.3, 0.31, 0.312, 0.322, 0.331  
 0.04, 0.4, 0.412, 0.424, 0.49

**Reading and Interpreting Scales on Measuring Instruments**

## Questions P267

3ml, 12ml and 23ml	6g, 24g and 31g
0.8L, 1.3L and 2.4L	0.07g, 0.23g and 0.39g

**Time**

## Questions A P269

Eleven o'clock, 11:00 or 23:00	Half past seven, 07:30 or 19:30	Quarter past two, 02:15 or 14:15
Twenty past twelve, 00:20 or 12:20	Ten past three, 03:10 or 15:10	Quarter to four, 3:45 or 15:45
Twenty seven min- utes to eight, 07:33 or 19:33	Twelve min- utes to one, 00:48 or 12:48	Eight min- utes past ten, 10:08 or 22:08

Questions B P270

1)



⚠ image331.emf

2)



3)



4)



⚠ image335.emf

5)



6)



7)



8)



⚠ image340.emf

⚠ image341.emf

9)



⚠ image343.emf

**Number Lines**

Questions P272

100, 420 and 580

6, 29 and 54.5

0.7, 2.1 and 5.55

0.09, 0.35 and

0.535

0.4, 1.65 and 2.7

4, 9 and 22.5

## Chapter 36

# Glossary

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**Place value** — The value of a digit according to its position in the number — its column of units, tens, hundreds, and so on.

## Chapter 37

# Topics by grade

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**Grade 1** — Greater Than and Less Than Notation; Ordering Integers; Ordering Decimal Numbers; Reading Scales on Measuring Instruments; Time; Number Lines

**Grade 2** — Rounding to Decimal Places; Ordering Negative Numbers; Factors of a Number; Multiples of a Number; Doubling and Halving; Inverse Operations; Fractions of a Whole; Fractions of Shapes; Equivalent Fractions; Simplifying Fractions by Cancelling Down; Changing Mixed Numbers to Improper Fractions; Changing Improper Fractions to Mixed Numbers; Understanding Ratio; Ratio and Fractions; Proportion; Place Value of Whole Numbers Grade 2; Place Value of Decimal Numbers Grade 2; Multiplying Whole Numbers by Powers of Ten Grade 2; Dividing Whole Numbers by Powers of Ten Grade 2; Multiplying Decimal Numbers by Powers of Ten Grade 2; Dividing Decimal Numbers by Powers of Ten Grade 2; Rounding to the Nearest 10, 100 and 1000; Adding Whole Numbers Mental Method; Adding Whole Numbers Written Method; Subtracting Whole Numbers Mental Method; Subtracting Whole Numbers Written Method; Adding Decimal Numbers Mental Method; Adding Decimal Numbers Written Method; Subtracting Decimal Numbers Mental Method; Subtracting Decimal Numbers Written Method; Multiplying Using the Times Tables or By Counting On; Multiplying Whole Numbers by Single Digits; Multiplying Whole Numbers by Two Digits Column Method; Multiplying Whole Numbers by Two Digits Table Method; Dividing Using the Times Tables or By Counting On; Dividing Whole Numbers by Single Digits; Dividing Whole Numbers by Two Digits Chunking Method; Multiplying Decimal Numbers with One Digit; Negative Numbers on a Thermometer; Factor Spiders; Fraction Names; What is a Decimal?

**Grade 3** — Place Value; Rounding to Significant Figures; Estimating Using One Significant Figure; Adding and Subtracting Negative Numbers; Multiplying and Dividing with Negative Numbers; Square Numbers and Square Roots; Cube Numbers and Cube Roots; Indices and Roots; Common Indices or Powers; Reciprocals; Divisibility; Standard Index Form; Prime Numbers; Prime Factors of a Number; Prime Factors using Factor Trees; Highest Common Factor (HCF); Lowest Common Multiple (LCM); HCF and LCM from Prime Factors; Multiplying and Dividing with Decimals; BIDMAS; Fractions of a Quantity; Ordering Fractions by Comparing to One Half or One Whole; Ordering Fractions by Using Equivalent Fractions; Adding and Subtracting Fractions; Adding and Subtracting Mixed Numbers; Multiplying Fractions; Multiplying Mixed Numbers; Multiplying Fractions and Whole Numbers; Dividing Fractions; Dividing Mixed Numbers; Dividing Fractions and Whole Numbers; Percentages; Decimal Numbers and Percentages; Percentages and Fractions; Decimal Numbers and Fractions; Percentages of Shapes; Percentages of a Quantity Non-Calculator Method; Percentage Increase and Decrease Non-Calculator Method; Percentages

of a Quantity Calculator Method; Percentage Increase Calculator Method; Percentage Decrease Calculator Method; Repeating Percentage Increase and Decrease Calculator Method; Percentage Profit and Percentage Loss; Mixed Percentage Questions; Reverse Percentages; Simple Interest; Sharing in a Given Ratio; Proportion Unitary Method; Exchange Rates; Using a Calculator; Ordering Fractions with the Same Denominator; Adding Fractions with the Same Denominator  
**Grade 4** — Multiplying and Dividing Numbers with Indices; Ratio Questions with Algebra; Upper and Lower Bounds

## Chapter 38

# Further Reading

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### 38.1 Books in this series:

